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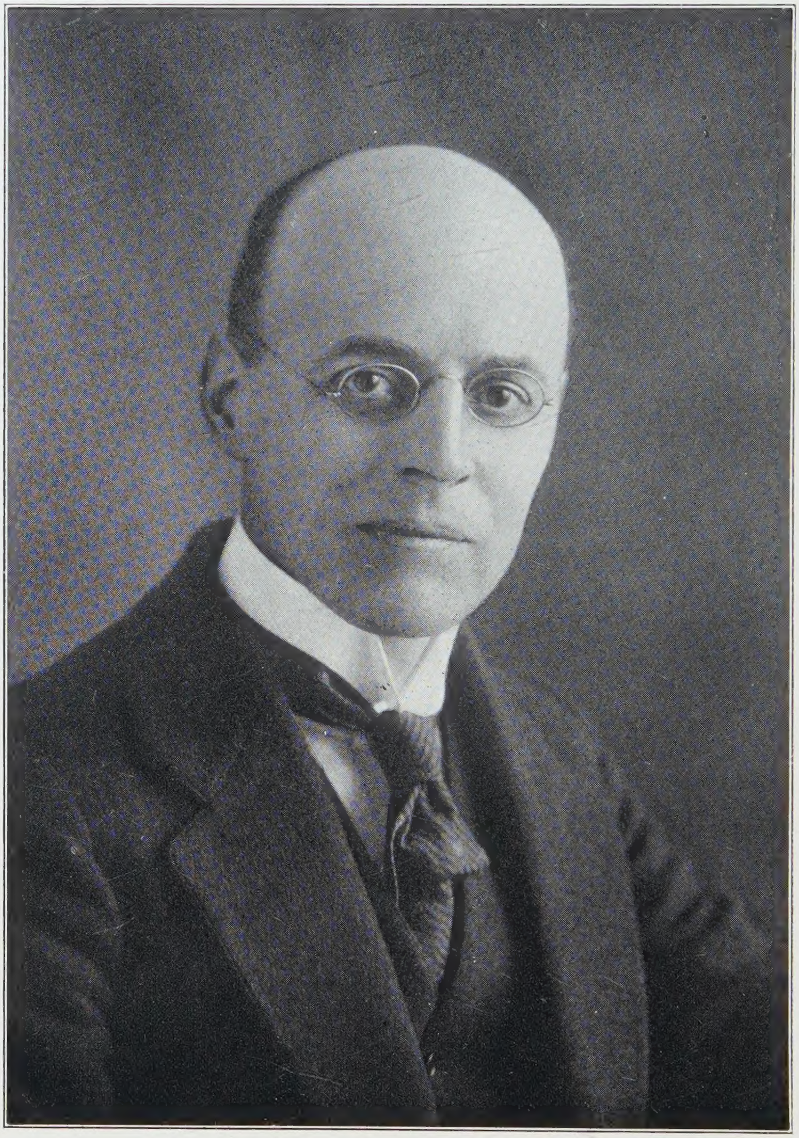

O. W. Richardson.

PRESIDENT, 1926–1928.

XIII.—THE PRESENT STATE OF ATOMIC PHYSICS.

(*Presidential Address delivered on February* 11, 1927, *by* Prof. O. W. RICHARDSON, M.A., D.Sc., F.R.S.)

§ 1.

THE discovery of the electron by J. J. Thomson at the close of the last century, followed by Planck's solution of the contradictions involved in temperature radiation phenomena by proposing the quantum theory, and later by Bohr's theory of the behaviour of the nuclear atom, to which Rutherford had been led by his studies of α particle deflections, have resulted in an insight into the physics of atoms which would have been incredible fifty years ago. They have turned spectroscopy from a heterogeneous collection of miscellaneous facts into a rapidly advancing science, and have created several other new branches of science of varying degrees of importance. Whilst these ideas suitably grafted on to the general body of earlier physical doctrine have been able to co-ordinate vast stores of existing facts, and to predict, perhaps, even vaster stores of new ones, they do not appear to be sufficiently wide in scope to embrace all that is required in atomic physics. There are certain groups of phenomena or facts which do not fall into the general framework which I have indicated, or, if they are forced in, the result is found to be a misfit. It would be very interesting to discuss these obdurate phenomena in some detail, but I shall have to content myself with doing little more than making a list of some of them. It is as follows :—

(1) The contradiction between the quantum theory and the wave theory of light. The contradiction is that the same light behaves as a wave when it is travelling through a telescope and is occupied with refraction or dispersion, and as a projectile when it is engaged in emission, absorption or photo-electric action. The energy of the radiation behaves as though it possessed at the same time the opposite properties of extension and localisation. I have pointed out the grave nature of this contradiction a good many years ago.*

(2) According to any reasonable interpretation of Bohr's theory, the spectra should be governed by the various kinds of quantum numbers which we should expect to take the various integral values 0, 1, 2, . . . ∞. But in actually working out the spectra spectroscopists find that they cannot in general get along with this set of numbers.† Half integral numbers insist on turning up. The case of band spectra furnishes very simple and convincing examples. On the theory (which is quite straightforward and fully borne out in other respects) the levels of the vibration bands should be $nh\nu_0$ (n=0, 1, 2, ...), where ν_0 is the fundamental vibration frequency at infinitesimal amplitude. As a matter of fact, they are found to be at $(n+\frac{1}{2})h\nu_0$ in the one case, the band spectrum of boron oxide, where the evidence is conclusive. There is no doubt of this matter, as the centre of the bands can be determined with certainty by using bands of isotopic molecules (cf., Mulliken, Phys. Rev., Vol. 25, p. 259, 1925).

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* Electron Theory of Matter, 2nd Edn., p. 507 (1916).

† *See* for example Sommerfeld. Three Lectures on Atomic Physics (1926).

(3) The Landé splitting factor, $g=1+\{j(j+1)+j_s(j_s+1)-j_a(j_a+1)\}/2j(j+1)$, determines the Zeeman effect in multiplets.[*] This is the formula required by the facts. The calculated formula is similar with the $j(j+1)$s replaced by j^2s. There are similar discrepancies between the calculated and found intensities of the components of multiplets.

(4) The remarkable effect discovered by Ramsauer[†] in Germany, and independently, but later, by Townsend and Bailey[‡] at Oxford, and by Chaudhuri[§] in my laboratory and working under my direction, each of these experimenters using a different method, that the mean free path of an electron in the inert gases, and especially in argon, becomes exceedingly long when the velocity of the electrons is reduced. This phenomenon seems incomprehensible on the Rutherford-Bohr picture of the atom. Dempster[||] has observed something similar with slowly moving charged hydrogen atoms in helium.

(5) If we contemplate a more complicated atom, the way in which the orbits interpenetrate and yet maintain their identity and stability seems very curious. One has to postulate a very harmonious intercoupling of the orbits. Even though this may be given a plausible justification on the quantum principles, this hardly destroys its mysteriousness.

(6) The calculated structures for the helium atom and the hydrogen molecule do not agree with the facts. It may be urged that these bodies furnish problems so complicated that the results must be regarded as endowed with an element of uncertainty; but it must at least be considered a matter for suspicion that all the calculations lead to ionisation potentials which are much too high.

(7) The magnetic anomalies. There are two of these—spectroscopic and gyromagnetic. According to the classical dynamics, as well as to the quantum theory, there is a general relation between the angular momentum j and the magnetic moment μ of any electron orbit—viz.,

$$j=\frac{2mc}{e}\mu$$

The gyromagnetic anomaly is that when a suspended rod is caused to spin by suddenly magnetising it along the axis, or if an unmagnetized rod is magnetized by spinning, the relation between the magnetic moment M and the angular momentum U is found, experimentally, to be

$$U=\frac{mc}{e}M$$

i.e., the factor has half the expected value. The spectroscopic anomaly is something similar.[¶] For the excited electron the proportion between what may be identified as the angular momentum and the magnetic moment has the normal value, but for the unexcited core (ground term) the magnetic moment is twice as large as would appear to correspond to the angular momentum. A collection of various proposals for explaining away the gyromagnetic anomaly has been given by Stoner.* I will add to it another which is a specialisation of one I have already given. Suppose the nucleus has a quantized spin with angular momentum equal in magnitude to one half of, and opposite in sign to, that of all the electrons in the atom.† To get some figures we need to make some hypothesis about the structure of the nucleus. Fortunately we only need the order of magnitude, so that I shall assume that we shall be somewhere near the mark if we treat the nucleus as a uniformly charged non-conducting spherical surface of radius R kept from exploding by a uniform non-electromagnetic tension. The magnetic moment of this structure is

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[*] *See* E. C. Stoner, "Magnetism and Atomic Structure," p. 238 (1926).

[†] Ramsauer, Ann. der Physik., Vol. 64, p. 451 (1921).

[‡] Townsend and Bailey, Phil. Mag., Vol. 43, p. 593 (1922).

[§] Chaudhuri, Phil. Mag., Vol. 46, p. 461 (1923).

[||] Dempster, Nature, Vol. 116, p. 900 (1925); Phys. Rev., Vol. 27 (1926).

[¶] cf., Sommerfeld. Three Lectures, p. 41.

$$M=\frac{eR^2}{3c}\omega$$

if ω is the angular velocity.‡ The moment of momentum is

$$U=I\omega=\frac{e^2R}{18\pi C^2}\omega$$

The mass of the sphere is $m=e^2/8\pi c^2R$ (electromagnetic)$+4/3\pi R^3\varepsilon/c^2$ (non-electromagnetic). The non-electromagnetic energy is $4/3\pi R^3\varepsilon$. If the electron forces are balanced by a normal non-electromagnetic tension $\varepsilon=e^2/32\pi^2R^4$, so that $m=e^2/6\pi C^2R$ and $M/U=e/cm$. If the suffixes a, e and n denote atom, electron and nucleus we have now

$$j_a=\Sigma j_a-j_n$$

$$\mu_a=\frac{e_e}{2m_ec}\Sigma j_e+\frac{e_n}{m_nc}j_n$$

By hypothesis $$\Sigma j_e=2j_n,\ \frac{e_n}{m_n}\div\frac{e_e}{2m_e}=10^{-3},\text{ about,}$$

so that $$j_a/\mu_a=\frac{\Sigma j_e-j_n}{\Sigma j_e}\cdot\frac{2mc}{e}=\frac{mc}{e}$$

as accurately as it has been measured.

(8) Space Quantisation and the Selection Rules. Whilst these can be presented as a fairly consistent body of doctrine which unquestionably expresses something very real in the actual phenomena, the physical interpretation seems to present considerable difficulties.

(9) The excitation of soft X rays.§ Whilst there is much experimental evidence of doubtful value in this field, some of the inflexions which seem to have no ascertainable connection with the Bohr scheme of levels are in my judgment well established experimentally.

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*Stoner, "Magnetism and Atomic Structure," p. 192. *See* also Richardson, Phys. Rev., Vol. 26, p. 248 (1908); 1921 Solvay Conference, loc. cit., and Proc. Roy. Soc., Vol. 102, p. 538 (1922).

† O. W. Richardson, Nature, Vol. 117, p. 652 (1926).

‡ O. W. Richardson, 1921 Solvay Physics Conference, p. 219; Gauthier-Villars, Paris (1923).

§ *See* Richardson and Chalklin, Proc. Roy. Soc., A, Vol. 110, p. 247 (1926); and K. T. Compton and C. H. Thomas, Phys. Rev., Vol. 28, p. 601 (1926).

(10) The peculiar distribution in angle of the electrons of medium speeds reflected by various solid elements discovered by Davisson.* These interesting phenomena also seem to lie outside the scope of the theories to which I have referred. There are other difficulties also concerning the detailed interpretation of spectra. These are mostly of an intricate character, and I shall not enter into them; in fact, I am hardly sufficiently familiar with them to be well fitted to do so.

§ 2. THE NEW QUANTUM MECHANICS.

Whilst the foregoing difficulties are by no means all of equal weight, and many of them if standing alone might be treated, perhaps, with indifference, yet taken together they form a solid obstacle. It is the consideration of such questions as these which in the last year or two has led atomic physicists to look for some radical departure from the points of view hitherto used by Bohr, Sommerfeld and their followers. The revolt, in fact, seems to have been started by Bohr himself.

The first attempt in this direction was made by taking the most obvious road. I do not, however, by this mean to imply that there was any really obvious road. What seemed solid in the situation were the two following things. The body of physical doctrine conveniently termed classical holds good for large scale phenomena where the frequencies involved are low. The physics of atoms in which very high frequencies are involved is governed by an entirely different body of doctrine, which has been found empirically, and is called the quantum theory. The distinction between a low and a high frequency is merely one of degree, not of kind, so that we should expect the quantum doctrine to extrapolate to the classical when the frequencies are sufficiently reduced, and this implies also a general correlation between the two. The groundwork of this was formulated by Bohr, and is known as the Correspondence Principle. This correlation is peculiar. For the solvable problems the co-ordinates in the classical motion (i.e., the description of an electron orbit) can be expressed as a trigonometrical series, such as

$$x = \sum_{p_1 p_2 \ldots p_k} A_{p_1 p_2 \ldots p_k} \cos \left[(p_1\omega_1 + p_2\omega_2 + \ldots + p_k\omega_k)b + a_{p_1 p_2 \ldots p_k} \right]$$

In this case the following electron "jumps" are possible,

$$n_1' - n_1'' = p_1 \quad n_2' - n_2'' = p_2 \quad \ldots \quad n_k' - n_k'' = p_k$$

and no others. Also the real frequency of the radiation emitted as a result of the jump is the average frequency (taken in a definite but very simple way) of the classical vibrations in a hypothetical classical motion from the n' to the n'' orbits. Also the intensity of the emitted radiation is correlated to the amplitude factor $A_{p_1 p_2 \ldots p_k}$ in the corresponding expression for the co-ordinate. In particular, if this is zero, that particular frequency and plane of polarisation will be absent in the emitted radiation. This is the essence of it, at any rate. For example, if it had been foreordained that the paths of the electrons in atoms were to be circles instead of Keplerian ellipses, we should have $x = A \cos(\omega t + a)$ simply. Since the only value of p which occurs here is 1, the only possible jump is $n' - n'' = 1$. Each spectral series would reduce to a single line. For example, Balmer's series would be replaced by $H\alpha(3 \rightarrow 2)$. Such an arrangement would have lightened the burdens of spectroscopists very considerably.

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* Davisson and Kunsman, Science, Vol. 54, p. 522 (1921); Phys. Rev., Vol. 21, p. 637; Vol. 22, p. 242 (1923).

I shall pass over a Paper by Bohr, Kramers and Slater,* which attempted to dispose of the contradiction between the quantum theory and the wave theory of light ((1) above), and another by Kramers† on the theory of dispersion, which helped to lead up to the further developments.

Whilst these remarks do not apply to these particular theories, it may be pointed out that some of the suggestions which have been made would introduce a new principle (or lack of principle) into physics by violating human free will. To account for the photoelectric effect, for example, it has been suggested that the emission of a quantum of radiation by an atom A and its subsequent absorption by an atom B involves a collusion between the atoms A and B of such a nature that the emission from B is predetermined by the event of ejection from A. Now suppose A is an atom on a star which the light left years ago, and B is an atom on a photographic plate of a telescope set up to photograph the star. It is obvious that there are a large number of things which I might do which would prevent that predetermination from being carried out. It is, of course, an indication of the seriousness with which the difficulties have been felt that such views should have received consideration.

The next step was by Heisenberg,‡ who made a kind of extension of the Correspondence Principle by pointing out that there was a very close analogy between the classical dynamics and the quantum dynamics of a peculiar kind. He pointed out that the things that we really know about atoms are the frequencies and amplitudes of the radiations they absorb and emit. We have no knowledge of such things as the position and time of revolution of an electron in an atom, and, in fact, these things may just be figments of our imagination. The key to what is real is probably to be found in the radiation. The properties of the radiation expressed in the corresponding classical and quantum theory forms are exhibited in the following table :—

Classical.	Quantum Theory.
$\nu(n, \alpha)=\alpha\nu(n)=\alpha\frac{1}{h}\frac{dE}{dn}$	$\nu(n, n-\alpha)=\frac{1}{h}\{E(n)-E(n-\alpha)\}$
$\nu(n, \alpha)+\nu(n, \beta)=\nu(n, \alpha+\beta)$	$\nu(n, n-\alpha)+\nu(n-\alpha,\ n-\alpha-\beta)=\nu(n, n-\alpha-\beta)$ or $\nu(n-\beta)(n-\alpha-\beta)+\nu(n, n-\beta)=\nu(n, n-\alpha-\beta)$
$A_\alpha(n)E^{\iota\omega(n)\alpha t}$	$A(n, n-\alpha)e^{\iota\omega(n, n-\alpha)t}$

n and α (or β) are two numbers which are integral in the quantum theory column, ν and ω indicate frequency, E energy and A amplitude. Any quantity $x\ (n, t)$ which is a function of n and t can be expressed classically by the Fourier expansions

$$x(n, t)=\sum_{\alpha=-\infty}^{+\infty} A_\alpha(n)e^{\iota\omega(n)\alpha t} \text{ if it is periodic, or}$$

$$x(n, t)=\int_{-\infty}^{+\infty} A_\alpha(n)e^{\iota\omega(n)\alpha t}\, d\alpha \text{ if it is not.}$$

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* Bohr, Kramers and Slater, Phil. Mag., Vol. 47, p. 785 (1924).

† Kramers, Nature, Vol. 113, p. 379 (1924) ; also Kramers and Heisenberg, Zeits. für Physik, Vol. 31, p. 681 (1925).

‡ Heisenberg, Zeits. für Physik., Vol. 33, p. 879 (1925).

According to Heisenberg, in the quantum theory $x(t)$ is to be taken as *represented* by the *totality* of all the values $A(n, n-\alpha)e^{\iota\omega(n, n-\alpha)t}$ and $x(t)$ squared as the totality of all the values $B(n, n-\beta)e^{-\iota\omega(n, n-\beta)t}$

$$= \sum_{\alpha=-\infty}^{\alpha=+\infty} A(n, n-\alpha)A(n-\alpha,\ n-\beta)e^{\iota\omega(n, n-\beta)t}$$

or the corresponding integral between $\pm\infty$ in the non-periodic cases. Heisenberg then asserts that if the quantities in the classical dynamics are translated in this way, and the operations of the classical equations are carried out according to the rules appropriate to the new type of quantities, the results form the answer to the corresponding problem in the quantum dynamics. For example, this method when applied to the non-harmonic oscillator, gives the energy levels as $(n+\frac{1}{2})h\nu_0$, as required by the band spectra, instead of $nh\nu_0$, as given by the older theory. An alternative independent development which has some advantages has been given by Dirac.*

The quantities which operate in this new quantum dynamics are not ordinary numbers, and the operations to which they conform do not agree in form with the laws of algebra. It was pointed out by Born and Jordan† that the laws which govern them are the same as those which govern the operations in the theory of matrices. The rules are framed so that the addition and subtraction of matrices are formally equivalent to algebraic addition and subtraction. Multiplication is defined as following the rows and columns rule of determinants. It agrees with the associative law for multiplication, but not in general with the commutative law—i.e., $(pq)r = p(qr)$; but pq is not in general$=qp$. On the other hand, $p(q_1+q_2)=pq_1+pq_2$. This identification with matrices has enabled a great deal of existing mathematics to be brought to bear on these problems, but it is a kind of calculus which I do not think that most physicists will find attractive. Anyhow, the real identification of the physical quantities of the quantum theory as matrices is very doubtful. What is really essential in this aspect of the matter is that $pq-qp$ should not be zero. This is practically the position taken by Dirac.

It will be seen that this theory of Heisenberg, Born, Jordan and Dirac is essentially and fundamentally one of discontinuities. I shall now pass on to the second way in which it has been attempted to widen the foundation of the older quantum theory, the so-called wave-mechanics. This method starts from the basis that all mechanics, including the quantum mechanics, is ultimately dependent on a wave-motion, and, in fact, that the solution of a mechanical problem is the solution of the partial *differential* equations of an appropriate wave-motion. This is evidently a theory whose basis is essentially one of continuity, and it is a most remarkable thing that the results of the two theories are *mathematically identical.* The foundations of this method are about 100 years old, and due to Hamilton. The Hamiltonian dynamics originated as a development of Hamilton's theory of optics, a fact which most people have forgotten.

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* Dirac, Proc. Roy. Soc., A, Vol. 109, p. 642 (1925); Vol. 110, p. 561; Vol. 111, p. 405; Vol. 112, p. 661 (1926); Vol. 113, p. 621 (1927).

† Born and Jordan, Zeits. für Physik., Vol. 34, p. 858 (1925).

§ 3. HAMILTON'S WAVE MECHANICS.

According to Fermat's principle, the track of the rays of light between any two fixed points is determined by the equation

$$\delta\int\frac{ds}{v}=0 \text{ (The principle of least time.)} \quad . \quad . \quad (1)$$

In general, the time should have an extreme value which may not in all cases be a minimum. In non-homogeneous media v will be a function of the co-ordinates xyz, through the relation $c/v=\mu(x, y, z)$, the refractive index ; so that the principle of least time may be written

$$\delta\int\mu(x, y, z)ds=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The variations δ have to be carried out in such a way that the terminal points are fixed.

For a conservative system in which the kinetic energy T is a homogeneous quadratic function of the generalised velocity components, the principle of least action takes the form

$$\delta\int 2Tdt=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For a single material particle this is the same as

$$\delta\int\sqrt{E-V}ds=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where E is the total and V the potential energy, and the total energy is understood to be the same in the actual as in the varied motion.

From (4) and (2) we see that the path of a moving particle in a space where the potential energy field is $V(x, y, z)$ is the same as that of a ray of light in the corresponding optical problem, provided

$$\mu(x, y, z)=\sqrt{E-V(x, y, z)} \quad . \quad . \quad (5)$$

Now we can work out geometrical optics either from Fermat's Principle or from Huyghen's Principle, and the results are the same. Turning to Fig. 1 we see that the successive wave fronts S_1, S_2, etc., are generated as the envelopes of infinitesimal spheres setting out simultaneously from every point of the preceding wave front and

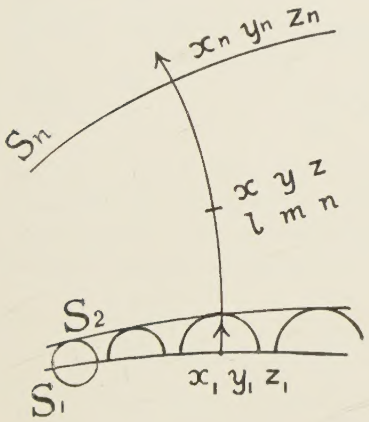

FIG. 1.

$$t_n-t_1=\int_{x_1y_1z_1}^{x_ny_nz_n}\frac{ds}{v}=\frac{1}{c}\int_{x_1y_1z_1}^{x_ny_nz_n}\mu(\text{l}dx+mdy+ndz)$$

where $ds=ldx+mdy+ndz$ is an element of the ray path. l, m, n and μ are functions of x, y, z, but the integral on the right can only depend on the terminal values, so that

$$t_n-t_1=\varphi(x_ny_nz_nx_1v_1z_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This equation is the equation of the successive wave-surfaces. In isotropic media the ray s is always perpendicular to the wave front, and thus for every displacement $\delta x_1\ \delta y_1\ \delta z_1$

$$\frac{\partial\varphi}{\partial x_1}\delta x_1+\frac{\partial\varphi}{\partial y_1}\delta y_1+\frac{\partial\varphi}{\partial z_1}\delta z_1=0,$$

provided $l_1\delta x_1+m_1\delta y_1+n_1\delta z_1=0$, where l, m, n are the direction cosines of the wave normal s_1 to S_1 at x, y, z, and therefore subject to

$$l_1^2+m_1^2+n_1^2=1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

It follows that

$$\frac{1}{l_1}\frac{\partial\varphi}{\partial x_1}=\frac{1}{m_1}\frac{\partial\varphi}{\partial y_1}=\frac{1}{n_1}\frac{\partial\varphi}{\partial z_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Similarly

$$\frac{1}{l_n}\frac{\partial\varphi}{m_n}=\frac{1}{m_n}\frac{\partial\varphi}{\partial y_n}=\frac{1}{n_n}\frac{\partial\varphi}{\partial z_n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and

$$l_n^2+m_n^2+n_n^2=1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The 6 equations (6) and (8) – (10) enable $x_ny_nm_nl_nm_nn_n$ to be ascertained when $x_1y_1z_1l_1m_1n_1$ are given. Thus the optical paths in the medium are completely determined by a single function $\varphi(x_ny_nz_nx_1y_1z_1)$ (Hamilton's characteristic function).

But it is implicit in (5) that the same result must hold for the path of a particle in a corresponding dynamical system. If α and β are undetermined multipliers, we have from (8) and (9)

$$d\varphi=\alpha(l_1dx_1+m_1dy_1+n_1dz_1)+\beta(l_ndx_n+m_ndy_n+n_ndz_n)$$

In going out from S_n along ds_n φ increases by

$$d\varphi=\frac{\mu_n}{c}(l_ndx_n+m_ndy_n+n_ndz_n)$$

and in going out from S_1 along ds_1 it diminishes by

$$\frac{\mu_1}{c}(l_1dx_1+m_1dy_1+n_1dz_1).$$

Thus

$$d\varphi=\frac{\mu_n}{c}(l_ndx_n+m_ndy_n+n_ndz_n)-\frac{\mu_1}{c}(l_1dx_1+m_1dy_1+u_1dz_1)$$

Putting

$$X_nY_nZ_n=\frac{\mu_n}{c}(l_n,\ m_n,\ n_n),\quad X_1Y_1Z_1=\frac{\mu_1}{c}(l_1,\ m_1,\ n_1)$$

we get

$$d\varphi=X_ndx_n+Y_ndy_n+Z_ndz_n-X_1dx_1-Y_1dy_1-Z_1dz_1$$

The quantities $X_nY_nZ_nX_1Y_1Z_1$ may be called the components of the normal *slowness* of the wave propagation.

We now proceed to the case in which S_n and S_1 are infinitely near wave fronts by putting $x_n=x_1+\lambda\delta t, y_n=y_1+\mu\delta t, z_n=z_1+\nu\delta t, X_n=X_1+p\delta t, Y_n=Y_1+q\delta t, Z_n=Z_1$

$+r\delta t$, $\varphi=W\delta t$ and $H=X\lambda+Y\mu+Z\nu-W$. We then find that if we regard H as a function of X, Y, Z, x, y and z,

$$dH=\frac{dx}{dt}dX+\frac{dy}{dt}\delta Y+\frac{dz}{dt}\delta Z-\frac{dX}{dt}\delta x-\frac{dY}{dt}\delta y-\frac{dZ}{dt}\delta z \quad \ldots\ldots\ldots \quad (11)$$

Thus

$$\left.\begin{array}{lll} \dfrac{dx}{dt}=\dfrac{\partial H}{\partial X} & \dfrac{\partial y}{\partial t}=\dfrac{\partial H}{\partial Y} & \dfrac{\partial z}{\partial t}=\dfrac{\partial H}{\partial Z} \\ \dfrac{\partial X}{\partial t}=-\dfrac{\partial H}{\partial x} & \dfrac{\partial Y}{\partial t}=-\dfrac{\partial H}{\partial y} & \dfrac{\partial Z}{\partial t}=-\dfrac{\partial H}{\partial z} \end{array}\right\} \quad \ldots\ldots \quad (12)$$

If XYZ are the components of the momenta and x, y, z of the displacements, these are precisely Hamilton's canonical equations. They are clearly the differential equations which correspond to (6) and (8)—(10) when the optical parameters are translated according to (5). Conversely, (6)—(10) are the integrals of (12).

The integral in (3) or (4) may be written in the form $\int \Sigma p_s dq_s$, where the ps and qs are written instead of X and x, etc., as the components of generalised momentum and displacement, this being a more usual notation. Keeping, however, to the case of a single particle, let the variation of the integral be carried out in a rather different way, so that two infinitely near paths are compared which take the same time t_n-t_1, but do not have the same end points or the same total energy E. The result is

$$\delta\varphi=\delta\int_{t_1}^{t_n}(p_1\dot{q}_1+p_2\dot{q}_2+p_3\dot{q}_3)dt=\Big/ p_1\delta q_1+p_2\delta q_2+p_3 d\delta_3\Big/_{t_1}^{t_n}+(t_n-t_1)\delta E \quad \ldots \quad (12.1)$$

whence

$$\left(\frac{\partial\varphi}{\partial q_1}\,\frac{\partial\varphi}{\partial q_2}\,\frac{\partial\varphi}{\partial q_3}\right)=+p_1p_2p_3 \text{ at } t_n \text{ and } =-p_1p_2p_3 \text{ at } t_1 \quad \ldots\ldots \quad (13)$$

and

$$\frac{\partial\varphi}{\partial E}=t_n-t_1 \text{ or } \varphi-E(t_n-t_1)=\text{const.} \quad \ldots\ldots\ldots \quad (14)$$

Since φ is a function only of $x_ny_nz_nx_1y_1z_1$ and E, this equation is of exactly the same form as (6), which was got directly from Fermat's principle. If instead of taking the function φ we take another function φ', which is derived by subtracting the total energy from the integrand in (12.1), we have (Hamilton's Principle)

$$\delta\varphi'=\int_{t_1}^{t_n}(p_1\dot{q}_1+p_2\dot{q}_2+p_3\dot{q}_3-E)dt=\Big/ p_1\delta q_1+p_2\delta q_2+p_3\delta q_3\Big/_{t_1}^{t_n}-E\delta(t_n-t_1) \,. \quad (15)$$

since

$$\delta\int_{t_1}^{t_n}Edt=\int_{t_1}^{t_n}\delta E dt+E\delta(t_n-t_1)$$

if E is constant for the natural (unvaried) path. From (15) it follows that

$$-E=\frac{\delta\varphi}{\delta(t_n-t_1)} \text{ or } \varphi'+E(t_n-t_1)=\text{const.} \quad \ldots\ldots \quad (16)$$

and

$$p=\text{grad } \varphi' \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

By constant in (14) and (16) we mean independent of the time.

Since grad $\varphi' = p = \sqrt{2m(E-V)}$ (compare $= {}^{n}5$), we can construct the surfaces φ', starting from any given value φ_0', by constructing the envelopes of the normals

$$\delta n = \delta \varphi_0' / \sqrt{2m(E-V)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

or after the manner of Huyghens' principle. Now, if the time varies, it follows from (16) and (18) that the system of surfaces will not vary, but the values of φ' will travel from surface to surface along the normals with velocity, in the space of equation (18), u, given by

$$u = E/\sqrt{2m(E-V)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

If, instead of supposing the surfaces to be fixed, we suppose them to wander so that each constantly takes the place of that which immediately preceded it, we get an equivalent picture which we may describe as a wave-motion in which the phase φ' travels with the velocity u. We may therefore call u the phase velocity. It is important to realise that this velocity is different from the velocity of the material particle. This is

$$v = (\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2)^{\frac{1}{2}} = [2(E-V)/m]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

All this is about a century old. The essence of it amounts to this: The motion of a particle in a field of force can be represented as a ray of a propagated wave surface. All classical dynamics is built up out of the motion of particles, and can therefore be equally well regarded as wave-motions with the energy travelling along certain rays, the paths of the particles. The correspondence holds on the optical side until the obstacles and apertures are diminished so that their dimensions become comparable with the wavelength. It does not include the diffraction group of phenomena.

§4. THE NEW WAVE MECHANICS.

A wave-theory of the motions of electrons in atoms has been developed in the last few years by L. de Broglie. The electron is regarded as a train of waves whose group velocity is shown to be v, the ordinary velocity of the electron, and whose phase velocity is C^2/v, where C is the velocity of light. I shall not follow de Broglie into the many interesting developments which he has made of these ideas. I shall, however, mention one of them which I believe to be of great importance. De Broglie makes this suggestion. Just as the corpuscular theory of light breaks down when the obstacles and apertures become comparable with the wavelength, so the particle theory of dynamics breaks down when the distances concerned are of atomic dimensions. And the reason is the same in both cases—namely, that both are wave phenomena, and in each case the dimensions concerned become camparable with the wavelength.

If we are to have a wave theory of mechanics valid down to atomic dimensions, one thing is clear—it will have to coincide with the Hamiltonian theory on the large scale. The way in which this extension is to be made is developed in a series of brilliant Papers by Schroedinger.* Let the wave-function be denoted by ψ. Then ψ will be of the form

$$\psi(x,y,z,t) = A(x,y,z) \sin \varphi'/K = A(x,y,z) \sin\left(-\frac{Et}{K} + \frac{S(x,y,z)}{K}\right) . \quad . \quad . \quad . \quad (21)$$

This is required by (16), since $\varphi'(= -Et +$a function of x, y, z only which we denote by $S(x, y, z)$) is a measure of the phase of the wave. Since Et and S have the dimensions of action, it is necessary to introduce the constant K in order to make the argument of the sine dimensionless. Now $A(x, y, z)$ represents the amplitude, and the frequency ν of the wave functions given by (21) is obviously

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* Ann. der Physik, Vol. 79, p. 361, p. 489, p. 734 ; Vol. 80, p. 437 ; Vol. 81, p. 109 (1926) ; Vol. 82, p. 257, p. 265 (1927). Phys. Rev., Vol. 28, p. 1049 (1926).

$$\nu=E/2\pi K$$

Schroedinger makes the very natural assumption that

$$K \text{ is a } \textit{universal constant}=h/2\pi, \qquad (22)$$

h being Planck's constant, which leads at once to the well-known universal relation between energy and frequency

$$E=h\nu \qquad (23)$$

The wavelength λ of the waves is

$$\lambda=\frac{u}{\nu}=h/[2m(E-V)]^{\frac{1}{2}}=h/mv \qquad (24)$$

Now for the nth Bohr circle of radius a_n we have $p=mva_n$ and $2\pi p=nh$, so that

$$a_n=nh/2\pi mv \qquad (25)$$

from which it is clear that h/mv is of the same order of dimensions as the atoms. This result is essential, because it is just at the atomic dimensions that the ordinary point-mechanics breaks down, and according to the ideas of de Broglie and Schroedinger it should break down when dealing with systems whose dimensions are comparable with the wavelength of the waves. This result also inspires confidence in the correctness of the assumption $K=h/2\pi$.

Equation (19) shows that the velocity of the waves depends on the frequency ν through the total energy E, so that the equation is an equation of dispersion. In the case of a single moving charged particle, the group velocity of the phase waves, which represent the motion, is equal to the actual velocity v of the moving particle.

On account of the breaking down of the ordinary mechanics, it is necessary to study the details of the wave-motion in dealing with atomic problems, and, as a preliminary to this, it is necessary to construct the equation of wave propagation. This process is in an experimental stage at present, the method adopted being to accept the simplest of any alternatives which offer themselves and to judge by the results. In the case of a single material point moving in an external field of force, Schroedinger takes the ordinary wave equation

$$\triangle^2\psi-\frac{1}{u^2}\frac{\partial^2\psi}{\partial t^2}=0 \qquad (26)$$

and puts for n the value given by (19). This depends on the space co-ordinates through V and on the frequency ν through E. The connection between E and ν restricts the use of equation (26) to such functions ψ as depend on the time only through the factor $e^{\pm 2\pi i}\frac{E}{h}t$. Thus $\frac{\partial\psi}{\partial t^2}=\mp\frac{4\pi^2E^2}{h^2}\psi$. If we put

$$\psi=e^{2\pi i}\frac{E}{h}t\,X(x,y,z)$$

we get from (19) and (26)

$$\triangle^2\psi+8\pi^2m(E-V)X/h^2=0 \qquad (27)$$

The wave function ψ is now to be derived by solving this partial differential equation, subject to such boundary conditions as may be appropriate to the problem.

Let us apply this to the hydrogen atom treated for simplicity as a single electron moving round a fixed inverse square centre of attractive force. Then $V=-e^2/r$, and in polar co-ordinates $r\theta\varphi$ (27) becomes

$$\frac{\partial^2 X}{\partial r^2}+\frac{2}{r}\frac{\partial X}{\partial r}+\frac{1}{r}\frac{\partial^2 X}{\partial \theta^2}+\frac{\cot\theta}{r^2}\frac{\partial X}{\partial \theta}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 X}{\partial \varphi^2}+\frac{8\pi^2 m}{h^2}\left(E+\frac{e^2}{r}\right)X=0 \quad . \quad (28)$$

The solution of this is sought in the usual way by putting $X=R(r)\times\theta(\theta)\times\Phi(\varphi)$, these functions each involving a single variable only, as indicated. With this substitution (28) can be broken up into three independent equations, and we find that Φ is the circular function $Ae^{in\varphi}+Be^{-in\varphi}$, θ is the spherical surface harmonic $P_m{}^n(\cos\theta)$, and $R(r)$ satisfies

$$\frac{d^2R}{dr^2}+\frac{2}{r}\frac{dR}{dr}+\left(\frac{4\pi m_1 E}{h^2}+\frac{4\pi m_1 e^2}{h^2 r}-\frac{m(m+1)}{r^2}\right)R=0 \quad . \quad . \quad . \quad . \quad . \quad (29)$$

($m=0$, 1, 2, 3, etc.). The limitation of n to whole numbers is necessary to make the dependence on the polar angles single valued. Now, Schroedinger has shown that the nature of the solutions of (29) depends very much on the value of the total energy E, which in this problem is quite definite, being equal to the sum of the kinetic energy of the moving electron and the potential energy $-e^2/r$. The only solutions of the equation (29) which are finite, continuous and single-valued throughout all space are those for which either

$$E>0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

or

$$E=-2\pi m e^4/h^2 l^2,\ l=1,\ 2,\ 3,\ 4,\ \text{etc.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Now the values (30) correspond to the hyperbolic orbits which are unquantized, and those given by equations (31) are the energy levels of Bohr's orbits. Actually, the functions which solve (29) are something like Bessel's functions, and are expressed by the following polynomial

$$X=x^m e^{-x}\sum_{k=0}^{l-m-1}\frac{(-2x)^k}{k!}\binom{l+n \text{ and}}{l-n-1-k},\ x=\frac{2\pi r}{h}\sqrt{-2m_1 E} \quad . \quad . \quad . \quad . \quad (32)$$

so that the wave function which represents the whole motion will be

$$\psi=\Sigma\{A_n\cos n\varphi+B_n\sin n\varphi\}P_m{}^n(\cos\theta)x^m e^{-x}\sum_{k=0}^{l-m-1}\frac{(-2x)^k}{k!}\binom{l+n \text{ and}}{l-n-1-k} \quad . \quad (33)$$

where l, m, n are integers which are subject to the requirements $n<m+1$ and $l>m$ (there is no solution for $l<m$, the integral values are required to make the solutions single valued, and the solutions are only valid if $n<m+1$). These are all purely mathematical requirements well established in the theory of the harmonic functions which satisfy vibration problems; but they have a remarkable resemblance to the discontinuous requirements which we are familiar with in the theory of quanta. According to (31), l must be identified with the total quantum number. It takes the values 1, 2, 3 . . . ∞. The values admissible for m or n are 0, 1, 2, 3 . . . ∞. If we simply identify the azimuthal number k with $m+1$, the value zero for this quantum number is obviously excluded, as the facts require. It is a well-known*

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* Byerly's Fourier's Series and Spherical Harmonics, p. 197.

property of spherical surface harmonics (tesseral harmonics) that there are $2m+1$ of them of the mth degree. Since $2m+1=2k-1$, this seems to explain the mystery that there are $2k-1$ Bohr levels associated with the azimuthal quantum number k. In Schroedinger's theory these numbers m and n give the nodes of the postulated θ and φ vibrations. It appears that X regarded as a function of x (equation (32)) has exactly $l-m-1$ real positive roots, so that the number of spherical nodes is equal to the radial quantum number. It is interesting to inquire where these nodes are. They are all within a few Ångstrom units of the centre of the atom. After passing the last zero in going outwards from the centre of the atom, the function X soon reaches a maximum or minimum, and then rapidly tends towards zero permanently. Thus, although the theory presupposes the vibration phenomenon to extend from the centre of the atom to infinity, it is for all practical purposes confined within a region which we are accustomed to regard as of atomic magnitude. This is something like the theory of the electromagnetic mass of an electron which distributes the mass all over the universe except inside the electron in theory; but it nevertheless locates it at the electron for all practical purposes.

It is interesting to inquire what is to be expected of an atom of hydrogen when in its state of lowest azimuthal quantum number ($k=1$ or $m=0$), the so-called *s* state. Since $n<m+1$, $n=0$ and the tesseral harmonic reduces to $P_0(\cos\theta)=1$, so that

$$\psi_{m=o}=\sum_{k=0}^{l-1} A e^{-x}\frac{(-2x)^k}{k!}\binom{l}{l-1-k},\quad x=2\pi r\frac{\sqrt{-2m_1E}}{h} \qquad . \quad . \quad . \quad . \quad . \quad (34)$$

This expression is independent of the angles φ and θ, so that the atom in an *s* state possesses spherical symmetry. This result has been tested by R. G. J. Fraser, who orientated the hydrogen atoms in a beam of canal rays by a magnetic field (space quantisation). He then sought for a difference in the mean free path for the orientated, as compared with the non-orientated, atoms. The result was entirely negative, which is in accordance with the requirements of Schroedinger's theory, and entirely opposed to any other theory of atoms which works.

It is important to recognise that equation (27) is restricted to cases of not more than 3 dimensions in the space co-ordinates, such as that of the hydrogen atom treated as a one body problem. In general Schroedinger* has shown that for a system whose Hamiltonian function is

$$\tfrac{1}{2}\sum_{l=1}^{n}\sum_{k=1}^{n} a_{lk}p_lp_k+V_{(q_1q_2\ldots q_n)},\quad a_{lk}=a_{kl} \qquad . \quad . \quad . \quad . \quad . \quad (27.1)$$

the equation has to be replaced by

$$\Delta p^{\frac{1}{2}}\sum_{l}\frac{\partial}{\partial q_l}\left(\Delta_p{}^{-\frac{1}{2}}\sum_{k} a_{lk}\frac{\partial\psi}{\partial q_k}\right)+\frac{8\pi^2}{h^2}(E-V)\psi=0 \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27.2)$$

where Δp is the determinant $|\Sigma\pm a_{ik}|$, when the frequency is restricted to E/h, or, if the frequency is unrestricted, it is to be replaced by

$$\Delta_p{}^{\frac{1}{2}}\sum_{l}\left(\Delta_p^{-\frac{1}{2}}\sum_{k} a_{lk}\frac{\partial\psi}{\partial q_k}\right)-\frac{8\pi^2V}{h^2}\psi\mp\frac{4\pi i}{h}\frac{\partial\psi}{\partial t}=0 \qquad . \quad . \quad . \quad . \quad . \quad . \quad (27.3)$$

These more general equations are necessary for most of the problems mentioned in the sequel.

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* Schroedinger, Phys. Rev., Vol. 28, p. 1064 (1926).

I will now recount, without going into details, the results of the calculations of a number of problems by the new wave-mechanics.

(*a*) The Harmonic Oscillator.—The E levels are

$$(n+\tfrac{1}{2})h\nu_0 \qquad n=0,\ 1,\ 2,\ 3.\ \ldots\ldots \quad (35)$$

instead of $nh\nu_0$.

(*b*) The Rigid Rotator with a Free Axis.—The E levels are

$$n(n+1)h^2/8\pi^2A \quad n=0,\ 1,\ 2,\ 3\ \ldots\ldots \quad (36)$$

instead of $n^2h^2/8\pi^2A$. Since

$$n(n+1)=(n+\tfrac{1}{2})^2-\tfrac{1}{4} \quad \ldots\ldots \quad (37)$$

the new formula is equivalent to half quantum numbers where the differences of the levels alone are concerned.

(*c*) The Diatomic Molecule.*—The result is the same as the ordinary theory, except that half quantum numbers appear throughout.

(*d*) The Hydrogen Molecule Ion H_2+.—It appears† that the lowest azimuthal quantum number which is admissible is $k=m+1=2$, and that the ionisation potential of H_2+ is exactly equal to that of the neutral H atom—i.e., $Rh=13.5$ volts (R being Rydberg's constant). By imagining the neutral hydrogen molecule to be dissociated into hydrogen atom ions and electrons in two alternative ways it is easy to show that

$$I_{\mathrm{H2}}+I_{\mathrm{H2}+}=D+2I_{\mathrm{H}} \quad \ldots\ldots \quad (38)$$

Here I_{H2}, $I_{\mathrm{H2}+}$ and I_{H} are the ionisation potentials of H_2, H_{2+} and H respectively, and D is the heat of dissociation of H_2. If $I_{\mathrm{H2}+}=I_{\mathrm{H}}$ it appears that

$$I_{\mathrm{H2}}=D+I_{\mathrm{H}}=D+13.5 \text{ volts} \quad \ldots\ldots \quad (39)$$

The reliable experiments fix the value of I_{H2} at 15.9 volts almost with certainty to 0.1 volt, which gives $D=2.4$ volts. The most reliable estimate of D is about 4.3 volts.‡ Whether this discrepancy is in the experimental value of D or in the calculations is uncertain. But, at all events, the fact that the estimated value of I_{H2} is pulled down a lot from the value 23.7 volts, calculated by Pauli on the old mechanics, is hopeful.

(*e*) The Stark Effect.—The levels agree exactly with those calculated by Epstein, which are known to agree well with the experiments. The calculated intensities also agree well with the experimental estimates. In the calculation of the intensities of components of the hydrogen lines in the Stark effect it is necessary to make the hypothesis that the complex quantity ψ multiplied by its conjugate complex quantity represents the density of electric charge in the generalized space of the problem. In the simple hydrogen case this is the actual charge density, but in the more complex cases the interpretation is more involved, and perhaps still uncertain.

(*f*) Dispersion.—It is to be remembered that in the present theory the terms E_m are regarded as representing real vibrations of frequency E_m/h, and the line frequencies of the emission spectra are equal to the differences of their frequencies.

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* Fues Ann. der Physik, 80, 367 (1926).

† Alexandrow, Ann. der Physik, 81, 613 (1926).

‡ cf., O. W. Richardson, Proc. Roy. Soc., A, Vol. 113, p. 414 (1926).

It results from the theory that any atom with characteristic values E_m, etc., exposed to radiation of frequency ν has excited in it vibrations of the frequencies $\frac{1}{h}E_m \pm \nu$ (2 for each characteristic value), and these combine with the permanent vibration of frequency E_m/h to absorb or emit the observed scattered and refracted rays of the required frequency ν. The actual dispersion formula is very similar to one based on considerations of the Correspondence Principle, brought forward by Kramers to bring the refractory subject of dispersion within the quantum fold.

(*g*) The Compton Effect.—This has been treated in detail by W. Gordon,* who calculates both the intensity and frequency of the scattered radiation. This calculation is difficult, but the relations between the direction and frequency of the scattered radiation and the direction and velocity of the moving electron, which were deduced by Compton by treating the problem mechanically as that of the impact of a light dart on an electron, have been obtained by Schroedinger in a rather simple but very curious way. It has been shown by Brillouin† that the reflection of light, of wavelength λ, by a sound wave of wavelength L, can be deduced by representing the sound wave as a crystal whose atoms are in planes L apart. The light is found to be reflected at an angle θ given by Bragg's law, for the first order, $2L \sin \theta = \lambda$. If now the moving electron is represented by the equivalent "wave of electric density" according to Schroedinger's‡ ideas, and this is replaced by the equivalent crystal (in the method of Brillouin), then Bragg's reflection law, for the first order, gives the facts of the Compton effect as stated already, the incident light replacing the incident light in the crystal reflection case. In both these problems the crystal must, of course, be treated as a moving crystal, and the Doppler effect taken into account.

Schroedinger makes the following claim for the advantages of the wave-mechanics :—

(*a*) The laws of motion and the quantum conditions are deduced simultaneously from one simple Hamiltonian Principle.

(*b*) The discrepancy hitherto existing in quantum theory between the frequency of motion and the frequency of emission disappears in so far as the latter frequencies coincide with the differences of the former. A definite localization of the electric charge in space and time can be associated with the wave-system, and this, with the aid of ordinary electrodynamics, accounts for the frequencies, intensities and polarizations of the emitted light, and makes superfluous all sorts of correspondence and selection principles.

(*c*) It seems possible by the new theory to pursue in all detail the so-called transitions which up to date have been wholly mysterious.

(*d*) There are several instances of disagreement between the new theory and the older one as to the particular values of the energy or frequency levels. In these cases it is the new theory that is better supported by experiment.

If the present account does not establish these claims with conviction, it is not because the claims are overstated, but because this account is necessarily very brief and imperfect.

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* Zeits. für Physik., Vol. 40, p. 117 (1926).

† L. Brillouin, Annales de Physique, Vol. 17, p. 88 (1923).

‡ E. Schroedinger, Ann. der Physik., Vol. 82, p. 260 (1927).

To this I would add that the theory seems hopeful in some other ways. I will enumerate some of them.

(1) It offers, at any rate in principle, a way of escape from the first difficulty in our list. A co-ordination of phase between the exciting radiation and the material from which a photo-electric electron is ejected may be necessary. The requisite co-ordination may be a rather rare occurrence. However attenuated light is, we get interference (G. I. Taylor's experiment). This and other familiar phenomena show that light is not really a dart. How, then, shall we explain the photo-electric effect (or light absorption) ? In the reverse process of recombination at the instant of interchange, the actual phase of the wave of the impinging electron will have to fit with the phase of the atom wave and with that of the emitted light in some appropriate way. In absorption or the photo-electric effect similar conditions will have to be worked backwards. The light may traverse many atoms before it comes to one in which the phases conspire in the right way. Owing to its high velocity of propagation, it does not give the atom much time to get into a suitable condition for interaction.

(2) The $\frac{1}{2}$ quantum numbers and the $j(j+1)$ factors referred to under (2) and (3) seem to come naturally out of this theory.

(3) I do not know that anyone has shown explicitly how the long free paths in argon come about, but there seems to be nothing in the theory which is abhorrent to them.

(4) The theory obviously gets rid of the difficulty about the orbital electrons intermingling, yet preserving their identities. What is supposed to be real in these consist of the modes of vibration of a pseudodynamical system.

(5) The calculated ionization potential of the hydrogen molecule seems still too high, although much lower than by the old methods. However, it is not certain that either the calculations or the experimental evidence are yet final in this matter. The neutral helium atom has not yet been calculated.

(6) It is too early yet to say whether it will account for the peculiar results found in the soft X-ray field and Davisson's results in electron reflection, but it seems in a general way to be the type of thing which is required for these phenomena.

Before concluding I should like to mention one or two difficulties in the way of the wave-mechanics. (1) In its present form it appears to offer no explanation of the atomicity of matter and electricity. (2) The way in which it uses the expression $-e^2/r$ for the potential energy in the hydrogen problem seems peculiar, since, according to the solution of the problem, the charge e is no longer at a distance r from the nucleus. In fact, in general, the theory seems to have to work with prescribed potentials given by the ordinary theory, and not with the potentials which it would itself deduce if it were a consistent theory. It is difficult to know what weight to assign to these objections at this stage. My own feeling is that, unless there is a substantial measure of truth in the theory, it will prove one of the most remarkable coincidences in the history of physics.

I wonder if I have removed that feeling of perplexity which I referred to at the beginning of my address. I do not expect I have. At any rate, I think you will probably agree with me that if the last thirty years has been the most wonderful in the history of physics, the next thirty promise to be equally interesting.

XIV.—ELECTRICAL POLARISATION IN SELENIUM CELLS AND THE EFFECTS OF DESICCATION.

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ABSTRACT.

The Paper describes measurements of the secondary E.M.F.s displayed by selenium cells and the apparently abnormal effect of illumination on the corresponding secondary currents. Both are shown to be probably due to an invisible water film in parallel with the selenium. That this film is the principal seat of the polarisation seems to be proved by the fact that the polarisation disappears almost completely with prolonged drying. On this basis a satisfactory quantitative explanation of the various effects is given. An important practical consequence of the desiccation of the cell is the large increase obtained in its sensitivity to light.

INTRODUCTION.

THE investigation to be described originated in the observation that certain selenium cells, which had been used previously by one of the authors for photophonic purposes, exhibited secondary E.M.F.s after the passage of electric currents through them. This phenomenon was in itself not surprising; other types of selenium cells have probably frequently been observed to have a similar property. The peculiarity of behaviour which attracted attention was the fact that the polarisation voltage established with the selenium in darkness was very largely *diminished* by illumination, with partial recovery when the illumination was cut off. Even when the terminals of the selenium cell in the polarised condition were connected together by a constant very high resistance, so that it was in fact behaving as a secondary cell and giving out a small but finite current, the effect persisted—i.e., the potential difference between the terminals was *reduced* by illumination. The conclusion was inevitable that the secondary current given out by the polarised selenium cell also *became less* under the action of light, in contradistinction from the well-known enhancement of current which the same selenium cell, in common with others, displayed under illumination when used in the ordinary way. Apparently the cells, carrying a current driven by an externally applied voltage, possessed what is usually known as positive light sensitiveness, whereas they were *negatively* light sensitive in respect of their own polarisation currents.

But in the course of closer examination of this surprising phenomenon it was discovered, almost accidentally, that the polarisation itself was profoundly modified by long exposure of the cell to the effects of phosphorus pentoxide. In these circumstances the capacity of the selenium cells to be polarised almost completely disappeared, and the electrical resistance greatly increased. It has been considered reasonable to attribute these changes to thorough desiccation, and the behaviour of the cells can be accounted for satisfactorily, except in one respect of relatively small importance, on the assumption that the polarisation observed was not due to the selenium itself, but to something in the nature of a water film, removable by persistent drying, in parallel with the selenium between the electrodes of the cell.

An important practical result of the investigation is the experimental fact, which accords with the theory put forward, that the light sensitiveness of the selenium cells is greatly increased by desiccation. Drying reduces the absolute conductance, but the *ratio* of the conductance under light action to that in the dark is much enhanced.

DESCRIPTION OF THE SELENIUM CELLS.

The selenium cells used were made to the design of Prof. H. Thirring, and the construction has been described by him.* The electrodes are of copper foil, arranged in the form of a minute condenser, the sheets being separated by mica. The latter is slightly recessed, and the selenium layer, about 0·01 mm. thick, is spread over the copper edges so as to fill the shallow interspaces. A circular area about 6 mm. in diameter is so covered, and protected by a superimposed sheet of transparent celluloid. A sheet of ebonite with a circular hole in the appropriate position limits slightly the surface of selenium exposed, and forms the front of the cell, which, however, is not hermetically sealed. The whole unit is in the convenient shape of a rectangular parallelepiped 4 cm. long, 2 cm. broad and 1 cm. deep, the face containing the selenium layer being 4 cm. by 2 cm. The electrodes are connected to two brass blocks, insulated from one another by ebonite, which form the 4 cm. by 1 cm. faces, and the cell is conveniently held for experimental purposes by insulated metal spring clips bearing on these surfaces and connected to suitable terminals.

The cells, as supplied, although of like dimensions and nominally of the same construction, vary considerably in electrical resistance. In complete darkness this is of the order 10 to 20 megohms. In common with other selenium cells, those under consideration vary considerably in conducting power during use, for reasons which sometimes remain obscure. The light sensitiveness also is a variable factor, differing from cell to cell, and changing in one and the same cell with progress of time and use. On the average it has a high value ; the current carried, for the relatively high voltages customarily applied (20 volts or so), is increased something like ten-fold by the illumination due to 100 candles at one metre.

PRELIMINARY EXPERIMENTS.

In order to observe the polarisation E.M.F. of the selenium cells it was necessary, because of their high resistance, to use an electrostatic method of measurement. A Dolezalek quadrant electrometer was therefore employed as a voltmeter in parallel with the terminals of the cell. The insulation was found to be sufficiently high for the purpose by measuring the rate of leakage of a mica condenser of known capacity. Under these conditions the insulation resistance proved to be approximately 15,000 megohms, and since the condenser used for this test was no doubt itself responsible for part of the leakage, it was safe to conclude that the insulation of the electrometer alone was in excess of the above value. Compared with this, even the high internal resistances of the selenium cells were negligible, and practically the full value of any polarisation E.M.F. was therefore recorded. As used, with a needle voltage of about 94, the sensitivity of the electrometer was such as to give approximately

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* Phys. Zeit., Vol. 21, p. 67 (1920), and Proc. Phys. Soc., Vol. 39, p. 97 (1926).

1 mm. scale deflection per millivolt. The motion of the needle was highly damped, so that practically no oscillation occurred, but the slowness of the motion prevented reliable readings being taken until one minute after a potential difference had been applied to the quadrants. Only slow variations of this difference could be properly observed.

The currents passing through the selenium cells due to applied voltages were measured by means of a 400 ohm moving-coil galvanometer originally giving a scale deflection of about 3·6 mm. for 10^{-8} ampere ; the same galvanometer was sometimes used also to observe the secondary currents in the selenium cells, which in these circumstances were virtually short-circuited by the galvanometer, the internal resistance of the selenium being relatively extremely high.

Only small voltages (of the order of 1 volt) were applied to the selenium cells in these experiments. There were two main reasons for this ; first, in order that the effects of possibly small polarisation on the current might not be obscured ; and, second, to avoid, if possible, the well-known increase of the conducting power of selenium which arises from the continued application of the considerable voltages customarily used. From both these points of view the choice of voltage made appears to have been correct.

The selenium cell under investigation was held in a clip with ebonite insulation, red fibre having previously been rejected on account of its relatively high conducting properties and liability itself to polarise. The whole was enclosed in a wooden box which could be rendered light-tight or opened for purposes of irradiating the selenium. Having applied a measured voltage to the selenium for a measured time, the current carried being observed meanwhile, the applied voltage was removed by breaking the circuit, and the electrometer immediately connected to the terminals of the selenium cell. Readings of the electrometer were then commenced as soon as possible, and continued for considerable periods of time, thus recording the decay of the polarisation E.M.F. Usually the polarisation of the selenium cell and the early subsequent electrometer readings were carried out with the cell in complete darkness ; then, at a chosen time, the enclosing box was opened, thus exposing the cell to illumination, for a period long enough to observe the accompanying fall of E.M.F. The box was then closed again and the subsequent growth of E.M.F. recorded.

All the selenium cells tested—some half-a-dozen—showed the effects to be described, although in markedly different degree. For a detailed investigation one was chosen for which the polarisation was large, and typical results for this are shown in Fig. 1. The upper curve in this diagram shows the decay of polarisation voltage, the selenium cell being all the time in darkness. The polarisation here is the result of the application of 1 volt to the selenium cell for one hour, during which the current in the selenium had fallen from a value in excess of $5{\cdot}9 \times 10^{-8}$ amp. to $4{\cdot}5 \times 10^{-8}$ amp. in the first 30 minutes, and had thereafter remained practically constant until the removal of the voltage. The first reliable reading of the electrometer, taken one minute afterwards, was equivalent to 0·269 volt—a value too high to insert conveniently in the diagram. This upper curve is included for purposes of comparison with the lower one, which relates to the effect of illumination on the polarisation E.M.F. of the same selenium cell on another occasion. The decay of the E.M.F. was here allowed to proceed until it was not too rapid, the cell having been previously polarised by the same voltage (1 volt) for the same length of time (one hour). At the time indicated by *A* the cell was exposed to Pointolite lamp illumination approximately equal to 400 $\frac{\text{candles}}{\text{metre}^2}$. One minute afterwards the small electrometer reading *B* was taken and the illumination immediately cut off. The subsequent changes of the E.M.F. are shown—first a substantial rise and eventually a resumption of the decay. The dotted curve *AC* shows by comparison with the upper curve what probably would have been the course of events if the cell had not been irradiated; for, although the decay curves were not strictly reproducible, they were approximately so and of the same general shape. It is noticeable that the recovery of the E.M.F. after the cessation of illumination is slow, just as is the recovery in the normal photo-electric effect of the cells; but it seems likely that after a sufficient time the recovery curve would coalesce with the continuation of *AC*.

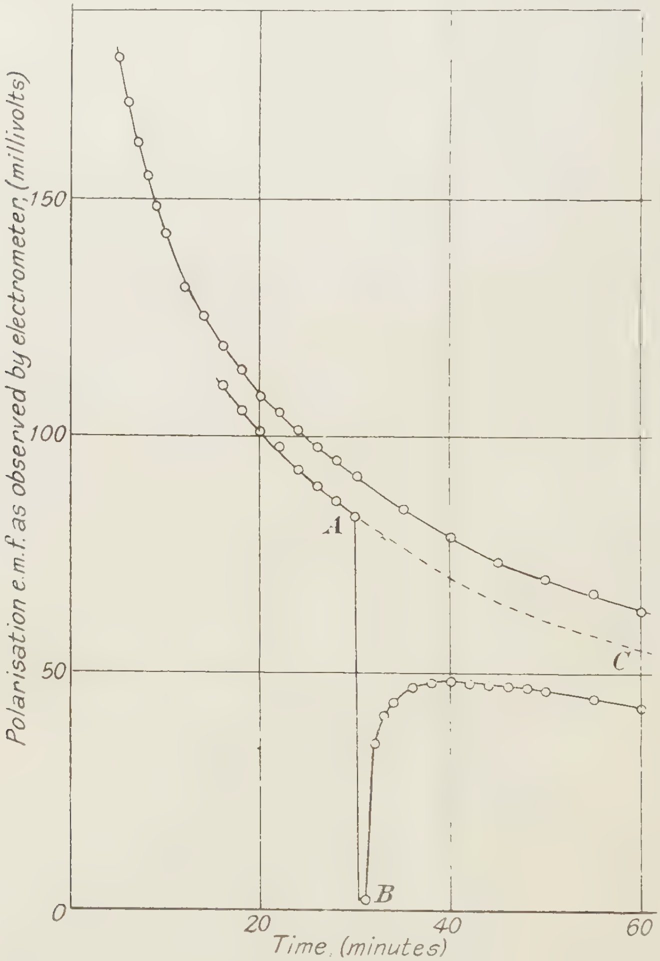


Fig. 1.—Upper Curve: Decay of Polarisation E.M.F. of Selenium Cell 466 in dark. Lower Curve: Effect of Illumination. 400 $\frac{\text{Candles}}{\text{Metre}^2}$.

The remarkable effect displayed by the portion *AB* of the curve is that the illumination causes a large *reduction* of the difference of potential between the terminals of the cell. Since the insulation was not absolutely perfect this must be interpreted as corresponding to a *reduction*, due to illumination, of the small current flowing in the circuit. In other words, the current through the selenium cell was apparently *diminished* by illumination when it was due to its own polarisation E.M.F., while the same cell, actuated by an externally applied E.M.F. gave, under conditions otherwise the same, the normal enhancement of current.

Experiments were also carried out with high resistances, of the same order as that of the selenium cell itself, in circuit with the cell, so as to cause the polarisation

E.M.F. to produce larger and definitely measurable currents. The electrometer readings were thus, of course, reduced approximately to one half, but qualitatively the effect of illumination was the same as before—namely, a large reduction of the difference of potential between the selenium cell terminals, and, by inference, of the current flowing through the external high resistance.

It was found also that the cells could be polarised while exposed to light, although the effect was revealed to a marked extent only after the illumination had been cut off. During polarisation under these conditions the current carried was, of course, much larger, but after the removal of the applied voltage, the cell still being illuminated, the polarisation E.M.F. measured by the electrometer was found to be very low. On darkening the cell, however, this E.M.F. rose to a maximum and eventually decayed after the manner of the curve in Fig. 1 subsequent to the point *B*.

The next step was to make current observations directly with the cell virtually short-circuited through the galvanometer immediately after polarisation in the manner already described. At this stage of the experiments the accuracy of the direct readings of current thus obtainable was low, since the sensitivity of the galvanometer was only sufficient to give readings of the order of a few millimetres at most. By the time the observed polarisation current had ceased to decay too rapidly—a necessary condition for satisfactory comparison—the galvanometer deflection would be only one or two millimetres. Under these conditions it was found that illumination produced *no observable effect* on the polarisation current. Subsequent measurements of a more accurate character, recorded later, have shown this not to be strictly true, but as a first approximation it is sufficient to indicate the great difference of the behaviour of the observed polarisation current obtained from the selenium cell according to whether the resistance external to it is high or very low.

The above are the main observations which led to a closer and more accurate investigation of the phenomenon, with a view to establishing quantitative relationships in what appeared to be a new photo-electric property of selenium. This work, now to be described, discovered instead the origin of the effects not to be in the selenium itself, which, apparently, is responsible for little, if any, of the observed polarisation.

Experiments with Improved Arrangement of Apparatus.

The new arrangement of the electrical circuit was fundamentally the same as before, permitting either the observation of the polarisation E.M.F. of the selenium cell on " open circuit " by means of the electrometer, or the polarisation current by means of the galvanometer which then virtually short-circuited the cell. A potentiometer was added to enable calibrations of the electrometer and galvanometer to be made from time to time as necessary, and to provide a range of voltages for application to the selenium cell. It was also found possible to increase the sensitivity of the galvanometer, which had a mirror of sufficiently good optical quality, by the expedient of increasing to more than 4½ metres the distance from the mirror to the observing scale. The sensitivity was thus raised to 16·4 mm. per 10^{-8} ampere, as compared with 3·6 mm. previously. This enabled the direct observations of current to be made much more accurately than in the preliminary experiments. It was also arranged so that the zero could be checked without disturbing the current through the cell. The zero actually remained remarkably constant.

The period and damping of the galvanometer suspension were such that a reliable current reading could be taken with a lapse of only five seconds after large changes of current, such as those involved by making and breaking the circuit. Fig. 2 shows a typical polarisation curve, obtained by applying 1 volt for one hour to a selenium cell, and then short-circuiting the cell through the galvanometer. The general behaviour appeared to be similar, although on a very much smaller scale as regards current, to that of an ordinary voltameter. An attempt was therefore made to find a polarising voltage small enough to be opposed completely by the back E.M.F., so that the current would be reduced to zero. Back E.M.F.s of the order of $\frac{1}{3}$ volt had been obtained with the application of 1 volt ; accordingly $\frac{1}{3}$ volt was applied to the same cell, and the diminution of current observed. It was found that the current fell from the value $1{\cdot}95 \times 10^{-8}$ amp. at 30 seconds after the application of the voltage to $1{\cdot}25 \times 10^{-8}$ amp. after half an hour, but that, although the voltage application was continued without intermission for approximately 45 hours, a current of $0{\cdot}92 \times 10^{-8}$ amp. still survived, and showed no tendency further to diminish. The back E.M.F. was thus found not to be a constant quantity, but to depend on and diminish with the applied voltage continuously. This was confirmed by a series of observations with a range of applied voltages between 0·33 and 1, in which electrometer measurements were made after polarisation until the current was steady. It was found that the voltage-time curves (of the type of the upper curve in Fig. 1) were all similar to one another—i.e., the ordinates at any time were in constant proportion, and that the polarisation voltages represented by these ordinates were roughly proportional to the voltage originally applied. This is seen in the following table :—

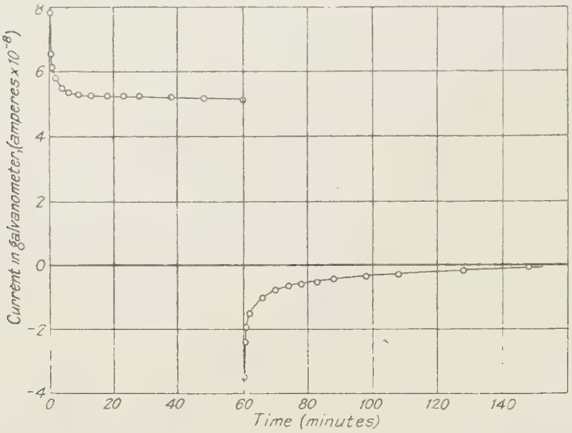


FIG. 2.—POLARISATION OF SELENIUM CELL 466 AS SHOWN BY GALVANOMETER VIRTUALLY ACTING AS SHORT CIRCUIT. ONE VOLT APPLIED FOR 1 HOUR ; THEN REMOVED FROM CIRCUIT.

TABLE I.

Applied voltage.	$\frac{\text{Polarisation voltage.}}{\text{Polarisation voltage due to 1 volt.}}$
1·00	1·00
0·70	0·65
0·60	0·57
0·50	0·52
0·40	0·39
0·33	0·37

The increased sensitivity of the galvanometric arrangement made possible a more accurate investigation of the effect of illumination on the polarisation current in the short-circuited selenium cell. The cell, having been polarised by 1 volt for one hour, was connected to the galvanometer and illuminated and darkened successively for equal periods at regular intervals, at first one minute, and later with longer intervening periods. The results are shown in Fig. 3, from which it will be seen that although the effect of illumination is small—quite negligibly so in comparison with the normal effect in selenium—the current is definitely increased by light. The increase, for the illumination specified, is about 10 per cent., compared with about 2,000 per cent. given by the cell under normal conditions.

Comparisons of one selenium cell with another showed that polarisation for equal periods produced effects of the same kind but widely different in magnitude. The course of the decay of polarisation E.M.F. was, however, similar in all cases, the curves being related by proportionality of ordinates at corresponding times.

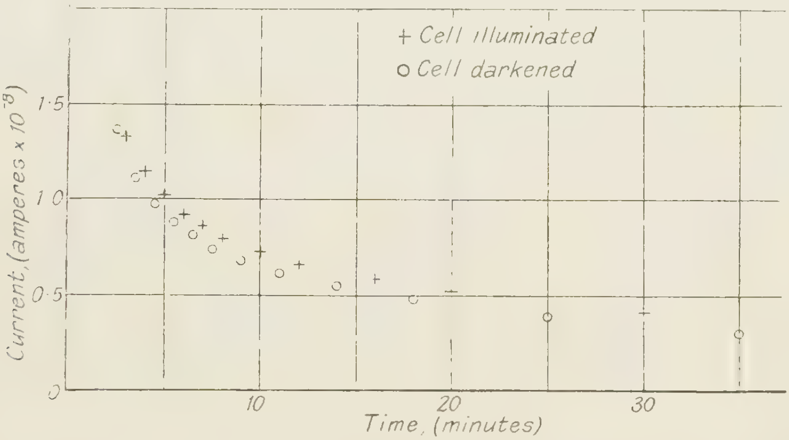


Fig. 3.—Effect of Illumination of 400 $\frac{\text{Candles}}{\text{Metre}^2}$ on Current through Galvanometer Short-circuiting Polarised Selenium Cell 466.

In obtaining the above results the importance of the effects of temperature had early been recognised as the possible cause of the observations not being strictly reproducible. Evidence was obtained that the current in the selenium cell with a constant applied voltage, and also the polarisation E.M.F. itself, increased rapidly with temperature. Steps were therefore taken to obtain more precise control over the temperature, so as to study the phenomena with more exactitude.

Effects of Desiccation.

The wooden box containing the selenium cell was itself placed in a metal enclosure surrounded by melting ice, 0°C. having been chosen as a suitable constant temperature for the continuation of the observations. In order to avoid the possible condensation of water on the cell and its connections, owing to the air becoming saturated, a tray of fused calcium chloride was introduced with the box. A constant temperature of 0·7°C. was reached in the locality of the selenium cell, and it was found that thepolarisation of the cell was much reduced, the back E.M.F. being little more than one-fifth of the previous value at atmospheric temperature (about 15°C.). This was naturally at first attributed to the temperature change, but even with the now constant temperature strict reproducibility of observations was still not possible. On allowing the cell to return to atmospheric temperature, while leaving it continuously in darkness and exposed to the drying agent, it was found that the cell had not recovered even approximately its former polarisation properties. The gradual reduction of current on applying 1 volt had almost disappeared; the actual current was diminished to about one-half; and the polarisation E.M.F., as indicated by the electrometer, had similarly diminished. Thus the effect of the drying agent was made evident.

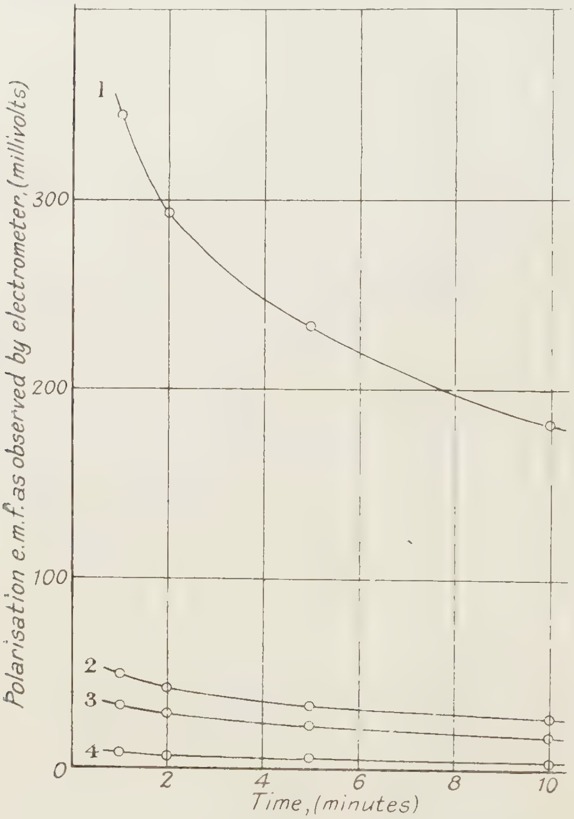


Fig. 4.—Polarisation of Selenium Cell 466—Effect of Desiccation.

Curve 1 Drying—none
„ 2 „ 6 days
„ 3 „ 10 „
„ 4 „ 33 „

At this juncture a specially designed and newly made container was brought into use. It is unnecessary here to describe it in detail. The essential features were as follows: It consisted of an air-tight vertical brass cylinder with an ebonite top fitted with a double glass window and carrying suitable insulated terminals to lead to the enclosed selenium cell and the thermo-couple used to determine the temperature of the cell. The cylinder was actually immersed as deeply as possible in a large water bath thermally insulated, but was also suitable for jacketing with steam, etc. The selenium cell was supported inside with its sensitive surface facing upwards and its temperature was ascertained by a thermo-couple, one element of which was inserted in a small brass block soldered to one of the brass clips which made the electrical contacts with the cell, and the other in the water bath.

This couple showed any small difference in temperature between the cell and the water bath, the temperature of the bath being read with a mercury thermometer graduated in 1/10°C. A vessel containing phosphorus pentoxide was placed at the bottom of the container, so that prolonged drying of its contents might progress. With the arrangement described it was found possible to maintain the temperature of the selenium cell constant at 15°C. to within $\frac{1}{3}°$ during a whole day.

With the selenium cell in darkness continuously and with the temperature adjusted very nearly to 15°C. during observations, readings were taken of the current due to the application of 1 volt and of the polarisation E.M.F. subsequent to one hour's application of this voltage. The readings extended over 33 days. Progressive drying was accompanied by a marked diminution of the actual current carried and the rapid disappearance of both the fall of current after the application of the voltage and the secondary E.M.F. of polarisation. The effects as regards the current are shown in Table II, and Fig. 4 shows the behaviour of the polarisation E.M.F. as measured by the electrometer.

TABLE II.—*Current Produced in Selenium Cell* 466 *by the Application of* 1 *volt During the Process of Desiccation.*

Duration of drying in days.	Current in 10^{-8} amp.			
	After 10 secs.	After 1 min.	After 10 min.	Steady value.
None	8·45	7·03	5·96	5·92
6	2·61	2·57	2·56	2·55
10	2·37	2·37	2·37	2·37
33	2·18	2·18	2·18	2·18

In the curves in Fig. 4 the ordinates are throughout in constant proportions to within the accuracy of the observations. These proportions, as percentages of the effects in the undried cell (Curve I) are given in Table III, which makes evident the rapid reduction of polarisation procured by drying, and points to the probability of its complete disappearance ultimately.

TABLE III.—*Variation of Polarisation E.M.F. in Selenium Cell* 466 *with the Period of Drying of Cell.*

Duration of drying in days.	$\dfrac{\text{Polarisation E.M.F.}}{\text{Polarisation E.M.F. in undried cell}} \times 100$
0	100·0
6	14·5
10	9·7
33	2·2

No simple relation has been found connecting these quantities; it is not, for example, an exponential one.

PROPERTIES OF THE DRIED CELL.

It was anticipated that the removal of the polarisation described, which was presumably attributable in some way to moisture of unknown, and perhaps varying amount, would result in the selenium cell becoming more constant in its electrical properties. This, to a certain extent, proved to be the case. Within certain limits of applied voltage the current was proportional to it, and did not vary from time to time for no apparent cause as had happened in undried cells. Provided the temperature was the same the current could be reproduced by applying the same voltage, and the cell displayed a definite resistance of about 46 megohms at 15°C. Above about 1·5 volts, however, Ohm's law ceased to be valid, the current becoming more and more in excess of that appropriate to this law as higher potential differences were used. At the same point, also, the well-known "enhanced conductance" effect began to appear—namely, that continued application of the voltage caused the current to grow with time. The apparent resistance of the cell became variable between wide limits, depending upon its previous history, and a long time was required for the cell to regain its primitive condition. Thus the property of the dependence of the conductance upon the duration of the current flow is assignable, in part at any rate, to the selenium itself, and is not wholly removed by desiccation. No constant resistance can be specified for the selenium cell except with a limitation as to the maximum current used. For the cell in question this maximum was

$$\frac{1{\cdot}5}{46\times10^{6}}=3{\cdot}3\times10^{-8} \text{ ampere approximately.}$$

Desiccation produced a striking enhancement of the light sensitivity of the selenium cell—that is, the ratio, with a fixed applied voltage, of the current in the cell under illumination to that in complete darkness was much increased. An exact figure for the ratio cannot, unfortunately, be given, since the need, now apparent, for precise measurements before desiccation was not foreseen. Moreover, a breakdown of the illuminating lamp during the progress of the work makes strict comparison still more difficult. The combined evidence of various pieces of experimental data points, however, to the conclusion that desiccation had increased the light sensitivity in the proportion of about 4 : 1, and that this increase was due almost entirely to the *reduction* of the "dark" conductance of the cell. This, it will be seen, is in accordance with the theoretical explanation of the general phenomena which it is now proposed to present.

PROPOSED EXPLANATION OF THE EFFECTS.

The effects which have been described can be accounted for satisfactorily on the assumption that something in the nature of a water film exists in the virgin selenium cell in such a way as to form a conducting path between the electrodes, in parallel with the selenium itself, and that the polarisation which is observed is due, in the main at least, to the development of a back E.M.F. in the water film during the passage of currents from extraneous sources. The exact localisation of the supposed water film, which, of course, is not visible, remains obscure; whether it is on the selenium itself, the celluloid cover or the ebonite insulating the electrodes, or, indeed, whether it is in the form of a film at all, has not been decided. But the almost complete disappearance of the polarisation with prolonged exposure to a drying agent, accompanied as it is by a reduction of the conductance of the cell, is in qualitative agreement with the previous existence of the film as the seat of the polarisation and the conductor of additional current. It is possible to go further, and, from available data, to obtain consistent quantitative agreement on the above assumptions.

For this purpose we may regard the virgin (i.e., undried) selenium cell as consisting of two conductors in parallel, first the selenium itself having a resistance S, and, second, a water film of resistance W, capable of developing a maximum back E.M.F. e_w as a consequence of the continued application of a definite voltage. In the thoroughly desiccated cell e_w may be regarded as having vanished, and W as having become infinite, leaving only S as the available conductor, in which there is little, if any, polarisation. On this basis, S has already been estimated to be 46 megohms.

In order to estimate e_w and W from the observations it is not possible to proceed directly, because the recording instruments—galvanometer and electrometer—were not capable of giving respectively the initial values of the total current upon application of the voltage and the potential differences between terminals of the cell upon the removal of the voltage. Further, the experimental curves are so steep in the regions of the first readings that extrapolations to time=zero would constitute little more than guesses. The curves only provide certain limiting values ; for example, that W is less than 16 megohms and that e_w is greater than 344 millivolts. There is, however, a definite relation between e_w and W, so that if one is known the other is determinate. Referring to the current values in Table II, the steady value of the total current in the undried cell, $5{\cdot}92\times10^{-8}$ ampere, is the sum of the currents in the selenium and in the water film. The former is $2{\cdot}18\times10^{-8}$ ampere, and the difference between this and the total current—namely, $3{\cdot}74\times10^{-8}$ amp.—passes through the water film under the action of 1 volt less the back E.M.F., which at this stage has become fully established. Thus

$$1\text{ volt}-e_w=3{\cdot}74\times10^{-8}\text{ amp.}\times W \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

From this point the procedure has been to choose values of W (and by consequence of e_w), and by trial and error find which gives the most consistent quantitative account of the various phenomena observed, while not being inconsistent with other checks such as those mentioned earlier. The value so obtained for W—namely, 12 megohms—and the corresponding value of e^w (550 millivolts) do in fact provide a quite satisfactory explanation of all the phenomena. Referring again to Table II, we may find the initial value of the total current by recognising that then there is no back E.M.F., and that the current in the water film is $\frac{1\text{ volt}}{12\text{ megohms}}=8{\cdot}33\times10^{-8}$ amp. The total current is this plus that in the selenium ($2{\cdot}18\times10^{-8}$ amp.), and equals $10{\cdot}5\times10^{-8}$ ampere, a value which is smoothly connected with the subsequent values. With regard to the electrometer readings (uppermost curve in Fig. 4), equation 4, which is developed later, gives the unobserved initial value as

$$V=\frac{550}{1+\frac{12}{46}}\text{ millivolts}=440\text{ millivolts.}$$

This again is in keeping with the general subsequent trend of the curve.

We have now to consider the effect of illumination on the system assumed when polarisation has been established and the current output from the system itself is being measured. In order on the one hand to preserve generality, and on the other to take account of a possible relatively small polarisation in the selenium itself, we shall suppose that a back E.M.F. e_s exists in the selenium. The system under consideration will thus be that represented in Fig. 5, where R is the resistance of the measuring instrument. Application of the current distribution laws gives the relation

$$C\left(1+\frac{R}{S}+\frac{R}{W}\right)=\frac{e_w}{W}+\frac{e_s}{S} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

where C is the current in the resistance R in which measurements are made. The effect of illumination may be supposed to have the single effect of reducing the value of S to a value S', according to the recognised property of selenium. In the actual experiments with the desiccated cell the ratio S/S' was found to be about 90 for the illumination used—namely, the equivalent of 100 c.p. at 50 cm. For purposes of calculation the values taken will therefore be

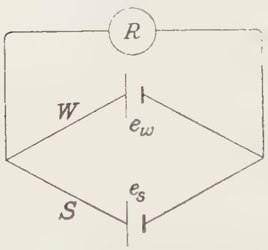


Fig. 5.—Schematic Representation of Undried Selenium Cell after Polarisation and under Observation with Polarisation Measuring Instrument of Resistance R.

$W=12$ megohms.
$S=46$,,
$S'=0{\cdot}5$,,

(*a*) *Measurements with galvanometer.*

In this case the value of R is 400 ohms—a quantity negligible in comparison with W, S and S'. Equation (2) therefore reduces to

$$C=\frac{e_w}{W}+\frac{e_s}{S}$$

It follows that if the selenium itself displays no polarisation—i.e., if $e_s=0$,

$$C=\frac{e_w}{W}$$

and the current is thus independent of the changes of resistance of the selenium accompanying illumination. This we have seen (Fig. 3) to be nearly true, and the main fact is therefore accounted for without assuming that the selenium polarises. The slight increase of current on illumination is, however, most easily explained by attributing to e_s a small value, which can be roughly calculated from the observations. (It should be noted that the actual current variation is in the opposite sense to that which the finite resistance of the galvanometer would produce—an effect which, in fact, is negligible.) If we suppose that the change of S to S' under illumination is the only photo-electric change, the current C' will be

$$C'=\frac{e_w}{W}+\frac{e_s}{S'}$$

and the current difference is given by

$$C'-C=e_s\left(\frac{1}{S'}-\frac{1}{S}\right)$$

According to the data represented in Fig. 3, the value of $C'-C$ is of the order of 10^{-9} ampere throughout, with a slight tendency to diminish with progress of time. The accuracy of the measurements scarcely justifies this variation being taken into account, and we may be content with obtaining the order of magnitude of e_s. With

$C'-C=10^{-9}$ ampere, and with the values of S and S' already given—namely, $S=46$ megohms, $S'=\frac{1}{2}$ megohm—we obtain

$$e_s=0{\cdot}5 \text{ millivolt approximately.}$$

This E.M.F. in the selenium, so small in comparison with that removed by desiccation and having its seat in the supposed water film, cannot be regarded as definitely established. It is too small to have been capable of definite direct measurement with the electrometer, the sensitivity of which was 1·6 millivolts per millimetre. But the definite, though small, change of current with illumination in the undried short-circuited cell has somehow to be accounted for, and the assumption of a relatively small polarisation in the selenium does this without being contradictory to the remaining experimental evidence.

(*b*) *Measurements with electrometer.*

In this case R becomes the leakage resistance of the electrometer, and its value is certainly more than 15,000 megohms. In comparison with R/W, R/S and R/S' unity is therefore negligible, so that equation (2) becomes

$$V=RC=\frac{\frac{e_w}{W}+\frac{e_s}{S}}{\frac{1}{W}+\frac{1}{S}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

V being the potential difference actuating the electrometer, now fulfilling the functions of a voltmeter. Neglecting for the moment the small quantity e_s, we have

$$V=\frac{e_w}{1+\frac{W}{S}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4$$

and V will evidently diminish with decrease of S, such as accompanies illumination. This is in accordance with the observations; moreover, the insertion of the data gives approximate quantitative agreement. For, clearly,

$$\frac{V'}{V}=\frac{1+\frac{W}{S}}{1+\frac{W'}{S'}},$$

V' being the terminal potential difference with the cell illuminated. Thus

$$\frac{V'}{V}=\frac{1+\frac{12}{46}}{1+\frac{12}{0{\cdot}5}}=\frac{1}{20} \text{ approximately.}$$

Of several measurements of the effect for the cell in question the most accurately observed was that the standard illumination reduced the electrometer indication from 144 millivolts to 8 millivolts, corresponding to a reduction ratio 1 : 18, which is not far from the ratio 1 : 20 indicated above.

A still closer agreement is obtained by allowing for the effect of e_s in the equation (3). According to this equation, the value $V=144$ millivolts corresponds to $e_w=180$ millivolts, the effect of e_s being quite negligible in this particular calculation. It becomes appreciable, however, under conditions of illumination when S has changed to the much lower value S'. The numerical value of V' is given by

$$V' = \frac{\frac{180}{12} + \frac{0 \cdot 5}{0 \cdot 5}}{\frac{1}{12} + \frac{1}{0 \cdot 5}} = 7 \cdot 7 \text{ millivolts},$$

and

$$\frac{V'}{V} = \frac{7 \cdot 7}{144} = \frac{1}{18 \cdot 6},$$

as compared with the observed ratio 1 : 18.

LIGHT SENSITIVITY.

The light sensitivity of a selenium cell is the ratio of its electrical conductance under exposure to standard illumination to that in complete darkness. The measurements of this sensitivity for the cell in question were made, both before and after desiccation, with applied voltages in comparison with which the polarisation E.M.F. was negligible. According to our theory, therefore, if we denote by L_1 the light sensitivity of the undried cell

$$L_1 = K \frac{\frac{1}{W} + \frac{1}{S'}}{\frac{1}{W} + \frac{1}{S}}$$

where K depends on the standard illumination chosen. The effect of drying being to make the conductance $1/W$ of the water film zero, the light sensitivity L_2 of the dried cell is given by

$$L_2 = K \frac{S}{S'},$$

The ratio

$$\frac{L_2}{L_1} = \frac{\frac{S}{W} + 1}{\frac{S'}{W} + 1} \quad \ldots \ldots \ldots \ldots \quad (5)$$

is clearly greater than unity, since S' is less than S. This simply means that the presence of the water film in the undried cell, giving rise as it does to an additional conductance which suffers no change with illumination, masks to some extent the photo-electric property of the selenium. Drying reduces the absolute conductance of the cell, but enhances the light sensitivity. Insertion of the available numerical data in equation (5) gives

$$\frac{L_2}{L_1} = \frac{3 \cdot 8 + 1}{0 \cdot 04 + 1} = 4 \cdot 6$$

which is sufficiently close to the roughly measured value 4 for the relative light sensitivities of the cell dried and undried.

This improvement of light sensitivity by desiccation is the most important practical result of the investigation. It may well be that the same process would procure similar advantages in other types of selenium cells.

POLARISATION IN THE DRIED CELL.

The proof provided by these experiments that the polarisation observed is almost entirely attributable to moisture does not exclude the possibility of a much more transient type of polarisation in the selenium-electrode system itself. The measuring instruments, owing to their deliberation of response, were incapable of giving information concerning what happens in the very early stages of the operations.

Some later experiments with the dried cell, which have had of necessity to be discontinued for the present, seemed clearly to indicate the existence of polarisation rapidly established on the application, and rapidly decaying on the withdrawal, of the voltage applied. The proof of this had to be indirect, and the method chosen was the customary plan of demonstrating a difference of apparent resistance according to whether direct or alternating current was used. Two commutators in phase on the same shaft were rotated at speeds which could be controlled. One caused periodic reversals of the current driven by 1 volt through the cell, and the other rectified this current for passage through the galvanometer. The steady direct current was first measured ; then the commutators were driven at a known speed and the corresponding alternating current noted. At 22 cycles per second the current in the cell was increased to 4½ times its D.C. value. Similar procedure with a non-inductive wire resistance of 1 megohm in place of the selenium cell resulted in a 25 per cent. reduction of current on causing it to alternate, an effect evidently due to contact inefficiency of the commutation. This imperfection of the apparatus prevented quantitative conclusions from being drawn ; but the experiments, nevertheless, seem to demonstrate quite definitely the existence in the dried cell of appreciable polarisation more short-lived than that dealt with in the main part of this Paper.

DISCUSSION.

Dr. A. B. **Wood**, in congratulating the authors, remarked that they had used Thirring cells, which are made on an improved design and are comparatively non-hygroscopic. He had found that cells of the Fournier d'Albe type, in which a coating of graphite on unglazed porcelain is divided by scratches bridged by selenium, are very hygroscopic. They could be freed from moisture by immersion in melted paraffin at 100°C., and in this way quite useless cells could be restored to activity, or cells with a dark/light resistance ratio of 4 : 1, for instance, would have their sensitivity increased up to a resistance ratio of 10 : 1.

Dr. D. **Owen** suggested that the cells should be hermetically sealed. Were they enclosed during the experiments, and, if so, in what material ?

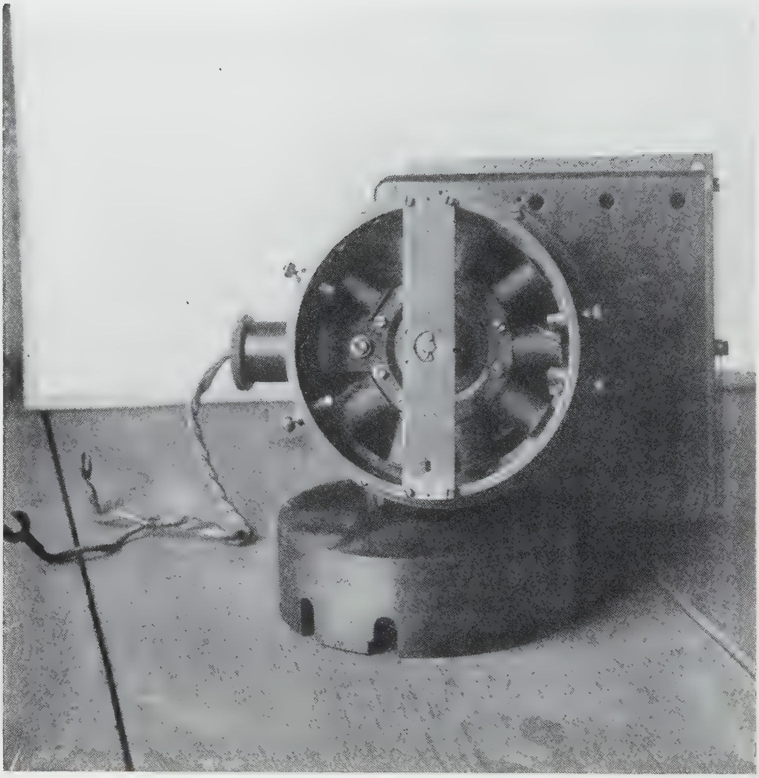
Dr. E. H. **Rayner** disagreed with Dr. Wood as to the non-hygroscopic character of Thirring cells. Although the mica used in their construction is a good insulator in itself, moisture is able to penetrate into the interspaces between the laminæ, forming a surface layer of appreciable conductivity.

Prof. E. N. **da C. Andrade** suggested that the cells might be sealed in glass vessels transparent to the illumination employed.

Prof. **Rankine**, in reply to the discussion, said that Thirring cells vary considerably in quality, that used in the present experiments being a specially selected one. Prof. Thirring covers the front surface of the cells with collodion, which seals them against moisture on that side, but the back surface, on which there is no selenium, is unprotected. In the present experiments the cell was at first roughly sealed, together with phosphorus pentoxide, in a brass cylinder with a glass window. Subsequently the cylinder was hermetically sealed, being, however, in communication with an air reservoir consisting of a toy balloon to allow for changes of pressure. The difference between the times taken in the two cases to reach a steady hygrometric state was considerable. The experiments had not been extended to cells of the Fournier d'Albe type, but it was interesting to hear that they could be improved by drying. It was desirable for manufacturers to seal the cells before putting them on the market, and no doubt the paraffin used by Dr. Wood acted as a seal.

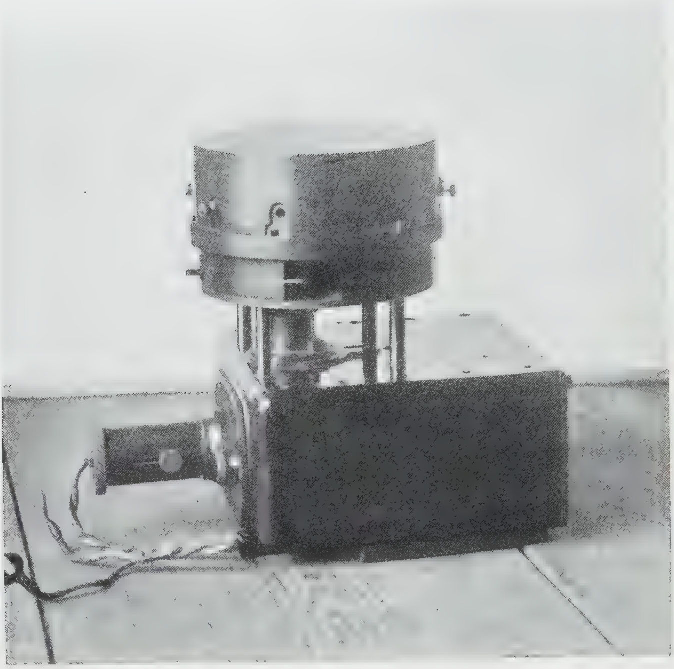
Dr. D. H. BLACK (communicated subsequently) : I have read the authors' Paper with great interest and I should like to ask one question. Have any measurements been made on the effect obtained by reversing the direction of the applied potential through the cell after the galvanometer deflection has reached a steady value ? If, as appears to be the case, the decrease of current with time is all due to the E.M.F. of polarisation which is set up, then on reversing the applied potential one would expect to obtain a steeper decay curve than the original, since the polarisation E.M.F. would then be in the same direction as the applied. If, however, there is a resistance change as well as a polarisation E.M.F., then it is quite possible for the reversal curve to lie below the original curve and to be of a different shape. I am at present engaged on some research on the electrical conductivity of oils, and I have studied, amongst other phenomena, the well-known decrease of current with time when a constant potential is applied across an oil film. Any attempt to account for all of this decrease by assuming a polarisation E.M.F. is rendered untenable by the shape of the current-time curve obtained by reversing the applied potential. The current on reversal starts at a low value, rises to a maximum, and then falls off to the same steady value as in the original case.

AUTHORS' reply to Dr. D. H. Black : No measurements were made on the effect of reversing the direction of the applied potential difference across the undried cell after it had been polarised. This rather obvious variation would have been of interest, and would probably have occurred to the authors, but for the fact of the disappearance of the polarisation effect just after the arrangements had been completed for measuring it more precisely under controlled conditions. Attention was thus diverted to the importance of the complete elimination of the cause of polarisation.

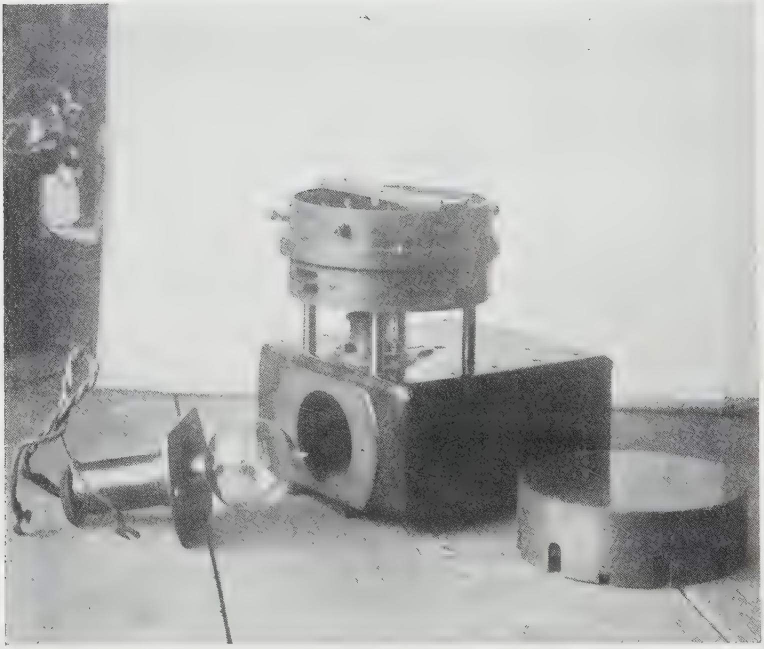


Plan of Motor Showing Polepieces.

[*To face page* 203.



Motor with Lamp and Cap Fitted.



Motor with Lamp and Cap Removed.

[*To face page* **203.**

XV.—SYNCHRONOUS ALTERNATING CURRENT MOTOR AND MECHANICAL VIBRATING SYSTEM MAINTAINED BY A THERMIONIC VALVE AT THE FREQUENCY OF THE VIBRATING SYSTEM.

By T. G. Hodgkinson, *M.C., A.M.I.E.E.*

Received December 2, 1926.

Abstract.

The design of a chronograph which will give a time trace subdivided into units and fractional units requires the association of a rotating system and a vibrating or time-keeping system.

If a low-frequency vibrating system be employed, the rotating system, although keeping good average time, may show exaggerated errors in unit subdivisions due to acceleration and deceleration between periods of control.

The Paper discusses the association of a high-frequency synchronous alternating-current motor with a high-frequency valve-maintained tuning-fork.

It deals with the design of valve-maintained motors and condenser-tuned valve-maintained tuning-forks and gives expressions for the frequency of the maintained system for motors coupled to tuning-forks both without and with tuning condensers.

In 1921 the writer coupled a 1,000 cycle synchronous alternating-current motor designed for the purpose to a valve circuit maintaining a 1,000 cycle tuning fork, and showed that for determined load conditions the tuning fork determines the frequency of the system, while the motor is maintained in rotation by the valve. This motor was used to drive a chronograph, and the instrument was shown at the Royal Society Soirée in May, 1923 ; but outside Service reports the method of control has not been discussed.

In this Paper the method of control of such a system is investigated as a circuit problem, so that design conditions are laid down, and for these conditions the difference between the true fork note and the frequency maintained by the system is determined.

This difference is small, and compares with the differences in fork circuits without other load discussed by S. Butterworth* and by the writer,† provided, of course, that the motor output is small compared with the energy capable of being stored by the vibrating system while vibrating within a reasonable amplitude limit.

Tuning fork frequency control of synchronous alternating-current motors (notably the Rayleigh phonic motor, the Leeds Northrup converter and similar machines) have been employed for many years. These devices employ local circuits, including vibrating contacts operated by an electrically maintained fork, which interrupt a steady electromotive force applied to the machine, or, alternatively, control a governing load in the machine circuit ; and the problem is of interest because the practicable use of vibrating contacts is limited to low-frequency schemes.

On the other hand, phase swinging of an alternating-current motor in a low-frequency scheme may cause serious time errors in a recording device driven by the motor, so that the use of higher frequencies becomes imperative for more accurate measurements.

The phase swing of a 1,000 cycle motor can cause only a relatively small time error, even if the swing is a large one ; hence the reason of the writer's original effort, which was to produce a neat, accurate chronograph employing only one valve ;

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* Proc. Phys. Soc., Vol. 32, page 345 (1920).

† Proc. Phys. Soc., Vol. 38, part I (1925).

however, the principles laid down in this Paper are not limited to tuning forks or one type of synchronous motors, and the subject has been dealt with as embracing the coupling of vibrating systems with rotating systems generally.

(1) THE MOTOR, VALVE MAINTAINED WITHOUT TUNING FORK (FIG. 1).

The particular motor mentioned earlier in this Paper as driving a chronograph has its winding distributed over eight poles, connected by a yoke, while the rotor is a laminated silicon steel disc, having twenty teeth ; but for the purpose of this work can be represented by a single winding surrounding a magnet, with a rotor having only one tooth.

The single winding has inductance l_1 and resistance r_1, and the amplitude of the E.M.F. produced in it when the machine is running is $V_M = V\frac{\omega}{\omega_F}$ where V is the E.M.F. when the frequency is that of the tuning fork we intend to couple the motor up with.

For the purpose of running the machine up to synchronism with the fork, or, if desired, for use when running with the fork, another magnet and winding can be employed.

This is shown dotted in Fig. 1, where the motor alone is shown coupled with a valve, and this method of maintenance of the motor alone is conveniently discussed before proceeding to the discussion of the motor coupled with the vibrating system.

The second motor winding has inductance l_2 and resistance r_2, while the E.M.F. produced in it when the machine is running at the frequency of the fork is $\frac{\mu}{m}V$, where m is the valve magnification factor and μ is a factor depending on coil dimensions.

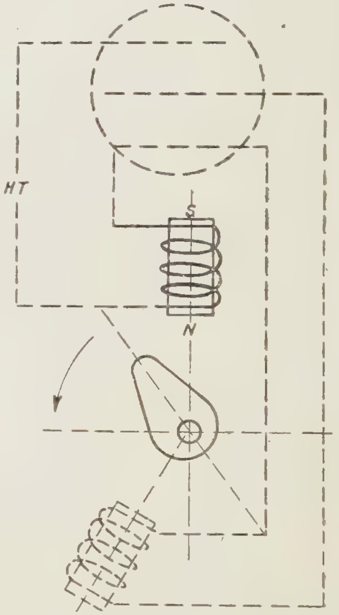


FIG. 1

This E.M.F. is arranged to lag behind the E.M.F. of the main motor winding by an angle θ, and is applied to the grid of the valve.

The main winding is included in the anode circuit of the valve, which has anode conductance s_1 and grid conductance s_2.

The anode current is i_1, the grid current is i_2, D is the time operator $\delta/\delta t$, and if we neglect all but the fundamental note the circuit equations are :

Grid Circuit.

$$(1/s_2+r_2+l_2D)i_2=\frac{\mu}{m}\frac{\omega}{\omega_F}V \quad \ldots\ldots\ldots\ldots \quad (1)$$

r_2 can easily be negligible compared with $1/s_2$, and it we neglect it here for the sake of tidiness, the E.M.F. in the valve grid is

$$V_g = 1/s\ i_2=\frac{\frac{\mu}{m}\frac{\omega}{\omega_F}V}{1+l_2s_2D}$$

Anode Circuit.

$$(1/s_1+r_1+l_1D)i_1=\left[\cos\theta+\sin\theta\frac{D}{\omega}+\frac{\mu}{1+l_2s_2D}\right]\frac{\omega}{\omega_F}V \quad . \; . \; . \; . \; . \; . \quad (2)$$

If the reaction of the motor in the grid circuit is

$$\frac{\mu}{m}\frac{\omega}{\omega_F}V\sin\omega t$$

the resultant E.M.F. in the anode circuit is

$$V_R=[\sin\overline{\omega t+\theta}+\mu\cos\delta\theta\sin\overline{\omega t-\delta\theta}]\frac{\omega}{\omega_F}V \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

and the current in the anode circuit

$$i_1=\cos\gamma[\sin\overline{\omega t+\theta-\gamma}+\mu\cos\delta\theta\sin\overline{\omega t-\delta\theta-\gamma}]\frac{\omega}{\omega_F}\frac{Vs_1}{1+r_1s_1} \quad . \; . \; . \; . \; . \quad (4)$$

where $\tan\delta\theta=l_2s_2\omega$ and $\tan\gamma=\dfrac{l_1\omega}{1/s_1+r_1}$. If now $\dfrac{\sin\overline{\theta+\delta\theta}}{\mu\cos\delta\theta+\cos\overline{\theta+\delta\theta}}=\tan\varepsilon$, the power supplied by the valve

$$P_A=\frac{\mu\cos\delta\theta\dfrac{\omega}{\omega_F}Vi_1}{2}\cos\overline{\varepsilon-\gamma} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5$$

and the power supplied to the motor

$$P_M=-\frac{\dfrac{\omega}{\omega_F}Vi_1}{2}\cos\overline{\theta+\delta\theta-\varepsilon+\gamma} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

Part of the power P_M is used to accelerate the rotor. Say that $P_M=P_\omega+$the accelerating power, and that $P_\omega=P\dfrac{\omega}{\omega_F}$

This assumes that the power P_ω is proportional to the speed ω—i.e., that the load is solid friction, and that P is the running power at the fork note. Substituting in (6) and making the accelerating power zero,

$$2P\tan\gamma_F(1+\tan^2\gamma)=\frac{V^2s_1}{1+r_1s_1}[\mu\cos\delta\theta\sin\overline{\theta+\delta\theta}\tan^2\gamma-(1+\mu\cos\delta\theta\cos\overline{\theta+\delta\theta})\tan\gamma]$$

where $\tan\gamma_F=\dfrac{l_1s_1\omega_F}{1+r_1s_1}$

Write $\dfrac{4P}{V^2s_1}(1+r_1s_1)=K$

and

$$\frac{\omega}{\omega_F}=\frac{K}{-(1+\mu\cos\delta\theta\cos\overline{\theta+\delta\theta})+\sqrt{(1+\mu\cos\delta\theta\cos\overline{\theta+\delta\theta})^2+K\tan\gamma_F(2\mu\cos\delta\theta\sin\overline{\theta+\delta\theta}-K\tan\gamma_F)}} \quad . \; . \; . \quad (7)$$

As a matter of fact, this equation is a biquadratic, since $\delta\theta$ is variable ; but for our purpose is conveniently discussed in this form.

There are two positive roots giving values for $\frac{\omega}{\omega_F}$, the lower of which is unstable that is, the lower root gives the speed at which the motor will start, and the upper the speed to which it will accelerate and remain stable.

The anode E.M.F., $\mu\cos\delta\theta\frac{\omega}{\omega_F}V=\frac{\omega}{\omega_F}V_A$, is often a determined quantity in design, limited by the H.T. supply, and if we require to do the maximum work in the motor for this E.M.F. the maximum value of P_M requires $\theta+\delta\theta=\pi-\gamma$. Substituting in equations (5) and (6) above

$$P_A=\frac{V_A^2s_1}{2(1+r_1s_1)}\cos\gamma\left[\cos\gamma-\frac{\cos 2\gamma}{\mu\cos\delta\theta}\right]$$

$$P_M=\frac{V_A^2s_1}{2(1+r_1s_1)}\frac{\cos\gamma}{\mu\cos\delta\theta}\left[1-\frac{\cos\gamma}{\mu\cos\delta\theta}\right]$$

when the speed $\omega=\omega_F$ and the efficiency is

$$E=\frac{\mu\cos\delta\theta-\cos\gamma}{\mu\cos\delta\theta\,[\mu\cos\delta\theta\cos\gamma-\cos^2\gamma]}$$

This efficiency is a maximum when $\mu\cos\delta\theta=\sin\gamma+\cos\gamma$, and is unity when $\gamma=0$ and $\pi/2$ and 0·5 when $\gamma=\pi/4$.

As with all valve problems, maximum power output is not compatible with maximum efficiency. Maximum power output from the valve and motor requires $\mu\cos\delta\theta=2\cos\gamma$, and on this basis the power output is constant, $P_M=\frac{V_A^2s_1}{8(1+r_1s_1)}=\frac{P_A}{2}$, and the efficiency is 0·5 throughout the range of γ.

This elasticity of design possibilities is due to the fact that the value of θ, or the phase relationship between the back E.M.F. of the machine and the anode E.M.F., is a determined quantity, independent of load. However, when the motor is coupled to the fork this condition is nullified by the additional fork anode E.M.F., so that γ can safely be made equal to $\pi/4$, or adjusted to suit the coupling conditions.

With a non-conducting grid $\delta\theta=0$, and if we require the maximum value of P_M when $\omega=\omega_F$, and make $\gamma=\pi/4$, V_M will be made equal to $V_A/\sqrt{2}$, $l_1\omega$ equal to $1/s_1+r_1$, and θ adjusted to $3\pi/4$.

Substituting these values in equation (7)

$$\frac{\omega}{\omega_F}=\frac{K}{\sqrt{K(2-K)}}$$

This is the starting speed. The higher speed is determined entirely by the power limit of the valve system.

Notice that with the conditions determined above $\frac{\omega}{\omega_F}<1$ when $K<1$.

The maximum power is $\frac{V^2s_1}{4(1+r_1s_1)}=\frac{V_A^2s_1}{8(1+r_1s_1)}$

and the motor will accelerate if this power is greater than P; in other words, if $K<1$.

Guess $K=2/3$, then $\omega/\omega_F=1/\sqrt{2}$ for the starting speed. The upper limit is determined by the power supply, and of course by the fact that the grid can only be relatively non-conducting within well defined limits, and that the value of m (and so μ) is constant only over a limited portion of the valve characteristic.

For a conducting grid case, guess $\tan\delta\theta=0{\cdot}5\tan\gamma$. This requires $\tan\theta=-3$ for the maximum value of $P_M=P$ if $\gamma=\pi/4$.

The power supplied to the motor at the speed ω_F is

$$\frac{V^2 s_1}{4(1+r_1 s_1)}=\frac{V_A{}^2 s_1}{8(1+r_1 s_1)}$$

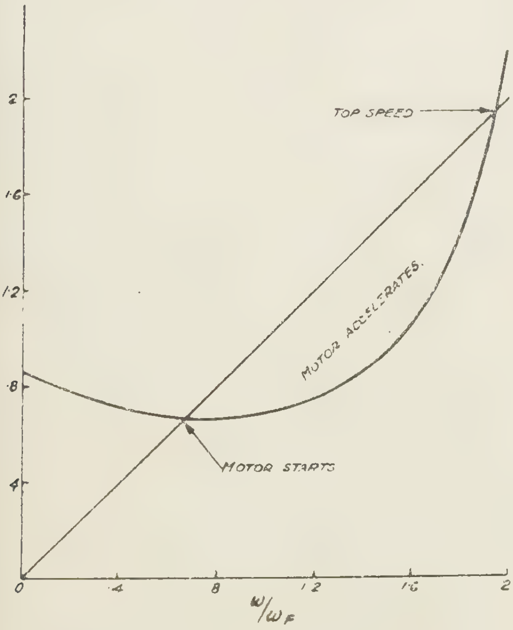
and the motor will accelerate if

$$\frac{V^2 s_1}{4P(1+r_1 s_1)}>1, \text{ or if } K<1 \text{ as before.}$$

In the Graph I, ω/ω_F is plotted against

$$\frac{K}{\sqrt{2}\sin\delta\theta\cos\delta\theta-1+\sqrt{(\sqrt{2}\sin\delta\theta\cos\delta\theta-1)^2+2K(2\cos^2\delta\theta-K)}}$$

assuming the above value for $\tan\delta\theta$ and making $K=0{\cdot}5$.



GRAPH 1.—SPEED SOLUTION FOR MOTOR AND VALVE (CONDUCTING GRID).

$$\frac{\omega}{\omega_F}=\frac{1/2}{\sqrt{2}\sin\delta\theta\cos\delta\theta-1+\sqrt{(\sqrt{2}\sin\delta\theta\cos\delta\theta-1)^2+(2\cos^2\delta\theta-1/2)}}$$

The curve represents values of right-hand side of the above expression for different values of θ, and the points of intersection with the straight line are the two solutions.

The motor will start when $\omega/\omega_F=0{\cdot}668$, and will accelerate until it is 1·94—i.e., between these values the power supplied to the motor exceed its demand at the speed.

If the motor is the determining factor, and we can do what we please with the valve and anode E.M.F. V_M, l_1, r_1, ω_1, and the power output required are fixed

$$P_M=\frac{V_M{}^2}{2l\omega}\sin\gamma\,(\mu\cos\delta\theta-\cos\gamma)$$

and the values of γ and $\mu\cos\delta\theta$ we have so far discussed may not meet the case if the value of $\dfrac{V_M{}^2}{2l\omega}$ is too small. For example, if $\mu\cos\delta\theta=2\cos\gamma$, the maximum power output requires $\gamma=\pi/4$, and is $V_M/4l\omega$, or if

$$\mu\cos\delta\theta=\overline{\sin\gamma+\cos\gamma},\ P=\frac{V_M{}^2}{2l\omega}\sin^2\gamma \text{ and is } \frac{V_M{}^2}{2l\omega},$$

when the valve conductance is very small and $\gamma=\pi/2$.

If we make

$$P_{\mathrm{M}}=\frac{V_{\mathrm{A}}^{2}s_{1}}{2(1+r_{1}s_{1})}\,\frac{B\sin\gamma\cos\gamma}{(B\sin\gamma+\cos\gamma)}=\frac{V_{\mathrm{M}}^{2}}{2l\omega}B\sin^{2}\gamma$$

the efficiency is

$$\frac{B}{B+\overline{1+B^{2}}\sin\gamma\cos\gamma}.$$

This efficiency is unity when γ is zero and $\pi/2$, and the minimum value (when $\gamma=\pi/4$) is less than 0·5, if B is greater than or less than unity.

If $B\sin^2\gamma$ is required greater than, say, 0·5, it will be better to make γ greater than $\pi/4$ by selecting a suitable valve conductance and then to adjust B to the required value.

Example : Say $B\sin^2\gamma$ is required$=1{\cdot}5$. If $\gamma=\pi/4$, $B=3$, and the efficiency is 0·375 ; on the other hand, if $\gamma=\pi/3$, $B=2$, and the efficiency is 0·48.

We have discussed how to design motors to suit valves, or valves to suit motors, and proceed to discuss the problems connected with the coupling of the motor and the fork.

There are various ways of doing this, and a typical method, with the fork windings in series with the motor windings, has been selected as embracing all of these if they are considered on an energy distribution basis. The method of investigation is to get out a circuit equation for the anode circuit, very large, although systematic for both the untuned and condenser tuned fork cases, and then to evaluate the difference between the true fork note and the maintained note, and the ratio between the work impedance (the A^2/β of Mr. Butterworth's Paper) and the inductive impedance of the fork windings, required to maintain the whole system, for a few special practical cases.

This conforms with the method of Mr. Butterworth's Paper and my own Paper quoted, and the solutions can thus be compared with solutions of the fork circuit above. The condenser tuned fork is also discussed here as a special case.

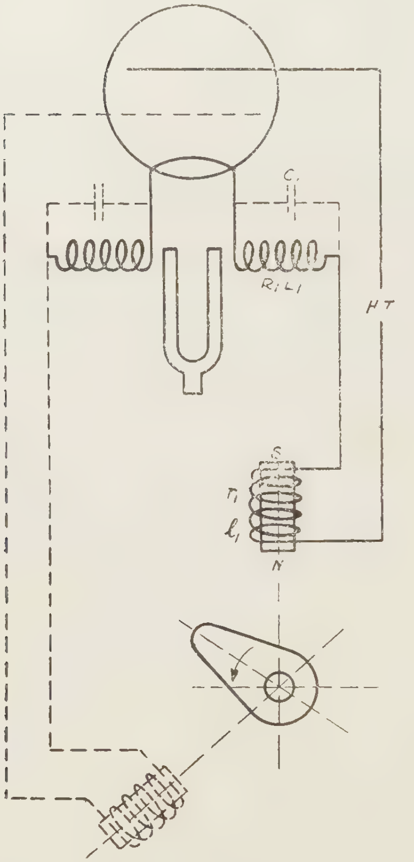


Fig. 2.—Motor Windings in Series with Fork Windings without Condensers.

Motor and Tuning Fork Valve Maintained at the Frequency of the Tuning Fork.

The motor anode winding is connected in series with the fork anode winding in the anode circuit of a valve, while the motor grid winding is connected in series with the fork grid winding in the valve grid circuit.

Notation—

Mass of fork supposed collected at end of prong	a
Frictional force per unit velocity	β
Control force per unit displacement	γ
Force in magnet gap per unit current in anode coil	A_1
Force in magnet gap per unit current in grid coil	A_2
Fork anode coil resistance and inductance and shunt capacity ...	R_1 L_1 C_1
Grid anode coil resistance and inductance and shunt capacity ...	R_2 L_2 C_2
Anode and grid conductances	s_1 s_2
Anode and grid currents	i_1 i_2
Valve magnification factor	m
Amplitude of back E.M.F. of the motor in the anode circuit when the machine is running at the fork frequency	V
Resistance and inductance of motor anode winding	r_1 l_1
Resistance and inductance of motor grid winding	r_2 l_2
Amplitude of E.M.F. of the motor in the grid circuit (if a grid winding is used) when the machine is running at the fork frequency	$\frac{\mu}{m}V$

Back E.M.F. of the motor in anode circuit

$$V_{\mathrm{M}}=\left(\cos\theta+\sin\theta\,\frac{D}{\omega}\right)\frac{\omega}{\omega_{\mathrm{F}}}V,\text{ where } D=\delta/\delta t$$

Back E.M.F. of the motor in grid circuit (if used)

$$V_0=\frac{\mu}{m}\,\frac{\omega}{\omega_{\mathrm{F}}}V$$

In this work I have continued to use the notation of my Fork Paper (quoted earlier, the essential fork notation of which was acquired from Mr. S. Butterworth's Paper (also quoted), so that the results of this Paper may be compared with those of the earlier Papers.

The *Grid Circuit* equation, ignoring the resistance of the grid motor winding, which is supposed negligible compared with the resistance of the grid electrode, is

$$(1/s_2+\overline{L_2+}i_2D)i_2+\frac{A_2D(A_2i_2-A_1i_2)}{aD^2+\beta D+\gamma}=V_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$V_g=\frac{i_2}{s_2}=\frac{V_0+\dfrac{A_1A_2Di_1}{aD^2+\beta D+\gamma}}{1+L_2+\bar{l}_2s_2D+\dfrac{A_2{}^2s_2D}{aD^2+\beta D+\gamma}}$$

The Anode Circuit.

$$(1/s_1+r_1+R_1+\overline{l_1+L_1}D)i_1+\frac{A_1D[(1+\overline{L_2+}l_2s_2D)A_1i_1-A_2s_2V_0]}{(1+L_2+l_2s_2D)(aD^2+\beta D+\gamma)+A_2{}^2s_2D} \quad . \quad . \quad (9)$$

$$=V_{\mathrm{M}}+m\frac{V_0+\dfrac{A_1A_2Di_1}{aD^2+\beta D+\gamma}}{1+\overline{L_2+l_2}s_2D+\dfrac{A_2{}^2s_2D}{aD^2+\beta D+\gamma}}$$

The power of the motor running steadily is $-\frac{\omega}{\omega_F}\frac{Vi}{2}\cos\varkappa$, where $\varkappa$ is the phase difference between V_M and i—i.e.,

$$P=-\frac{Vi\cos\varkappa}{2},\text{ or } V_M=-\frac{V^2}{2P}\cos\varkappa\left(\cos\varkappa+\sin\varkappa\frac{D}{\omega}\right)\frac{\omega}{\omega_F}i_1$$

$$\frac{V_M s_1}{1+r_1 s_1}=-\cos\varkappa\left(\cos\varkappa+\sin\varkappa\frac{D}{\omega}\right)\frac{\omega}{\omega_F}\frac{2}{K}i_1$$

$$\frac{V_0 s_1}{1+r_1 s_1}=-\frac{\mu}{m}\cos\varkappa\left(\cos\overline{\varkappa-\theta}+\sin\overline{\varkappa-\theta}\frac{D}{\omega}\right)\frac{\omega}{\omega_F}\frac{2}{K}i_1$$

since V_M is $\hat{\theta}$ in advance of V_0.

Substituting in equation (2) write

$$\frac{\overline{L_1+l_1\omega s_1}}{1+\overline{r_1+R_1}s_1}=\tan\gamma_1,\text{ and } \overline{L_2+l_2}s_2\omega=\tan\delta\theta_1,\text{ and}$$

$$\left[\begin{aligned}&\left(1-\frac{\omega^2}{\omega_F^2}+\frac{\beta}{\alpha\omega_F}\frac{D}{\omega_F}\right)\left\{\begin{aligned}&\left(1+\tan\gamma_1\frac{D}{\omega}\right)\left(\cos\delta\theta_1+\sin\delta\theta_1\frac{D}{\omega}\right)\\&+\frac{2}{K}\frac{\omega}{\omega_F}\cos\varkappa\left\{\begin{aligned}&\cos\overline{\varkappa+\delta\theta}+\sin\overline{\varkappa+\delta\theta}\frac{D}{\omega}\\&+\mu\cos\delta\theta\left(\cos\overline{\varkappa-\theta}+\sin\overline{\varkappa-\theta}\frac{D}{\omega}\right)\end{aligned}\right\}\end{aligned}\right\}\\&+\frac{A_1^2}{L_1+l_1}\frac{\tan\gamma_1}{\alpha\omega_F^2}\frac{D}{\omega}\left\{\cos\delta\theta_1\left(1-m\frac{A_2}{A_1}\right)+\sin\delta\theta_1\frac{D}{\omega}\right\}\\&+\frac{A_2^2}{L_2+l_2}\frac{\sin\delta\theta_1}{\alpha\omega_F^2}\frac{D}{\omega}\left[1+\tan\gamma_1\frac{D}{\omega}+\frac{2}{K}\frac{\omega}{\omega_F}\cos\varkappa\left\{\begin{aligned}&\cos\varkappa+\sin\varkappa\frac{D}{\omega}\\&+\frac{A_1\mu}{A_2 m}\left(\cos\overline{\varkappa-\theta}+\sin\overline{\varkappa-\theta}\frac{D}{\omega}\right)\end{aligned}\right\}\right]\end{aligned}\right]i_1=0$$

This expression of the motor E.M.F.s (rigid quantities), in terms of the varying current, phase and power, which have to adjust themselves to running conditions, is justified by the reservation that it requires that the system be working—i.e., that the motor is running in synchronism with and is supported by the fork.

Under these conditions the power supply to the valve is made such that the anode current and the fork amplitude are capable of growing to provide whatever power is required by the motor. The ratio K between the power required by the motor and $\frac{V^2 s_1}{4(1+r_1 s_1)}$ can have values greater than unity provided that the equations are satisfied, and that the fork amplitude is not allowed to grow excessive for its dimensions, and in subsequent examples values of K both above and below unity have been included.

Case (1).—Without Grid Reaction from the Motor.

If the grid conductance is negligible and the motor has no grid winding—i.e., there is no reaction from the motor, the solution we are concerned with if $\beta/\alpha\omega_F$ is very small—makes

$$1-\frac{\omega^2}{\omega_F^{\,2}}=\frac{\beta}{\alpha\omega_F}\,\frac{\tan\gamma_{1_F}+\dfrac{\sin 2\varkappa}{K}}{1+\dfrac{1+\cos 2\varkappa}{K}}\quad .\ .\ .\ .\ .\ .\ .\ .\quad (10)$$

and the condition equation is

$$\left(m\frac{A_2}{A_1}-1\right)\frac{A_1^{\,2}}{\beta(1/s_1+r_1+R_1)}\geqq 1+\frac{1+\cos 2\varkappa}{K}+\frac{\left(\tan\gamma_{1_F}+\dfrac{\sin 2\varkappa}{K}\right)^2}{1+\dfrac{1+\cos 2\varkappa}{K}}\quad .\quad (11)$$

Writing

$$\frac{\tan\gamma_{1_F}+\dfrac{\sin 2\varkappa}{K}}{1+\dfrac{1+\cos 2\varkappa}{K}}=F(x)$$

and

$$\left(m\frac{A_2}{A_1}-1\right)\frac{A_1^{\,2}}{\beta(1/s_1+r_1+R_1)}=N$$

we can rearrange our equation (9) for the anode circuit when the machine is running

$$\left[F(x)-\tan\gamma_{1_F}+(1-N+\tan\gamma_{1_F}\,F(x)\frac{D}{\omega}\right]i_1=\frac{\left(F(x)+\dfrac{D}{\omega}\right)V_M}{1/s_1+r_1+R_1}\quad .\ .\ .\ .\ .\quad (12)$$

The anode E.M.F. available for driving the motor is mV_g less the back E.M.F. of the fork in the anode circuit, and if the fork is large enough to store the energy, this E.M.F. can only change slowly, and is sensibly constant for any rapid change of load on the machine. If the current has reached a certain average value i_F, the anode E.M.F. is

$$\frac{(1/s_1+r_1+R_1)N\dfrac{D}{\omega_F}i_F}{F(x)+D/\omega_F}=\frac{N\dfrac{D}{\omega_F}V_M}{F(x)-\tan\gamma_{1F}+(1-N+\tan\gamma_{1F}F(x))\dfrac{D}{\omega_F}}$$

and the advance of V_M on this anode E.M.F. is

$$\theta_F=\tan^{-1}\frac{1-N+\tan\gamma_{1F}F(x)}{F(x)-\tan\gamma_{1F}}-\pi/2$$

Stable values of θ_F must make it greater than $\pi-\gamma_{1F}$—i.e., for stable running

$$\frac{N-(1+\tan\gamma_{1F}F(x))}{\tan\gamma_{1F}-F(x)}\geqq\cot\!\text{an}\,\gamma_{1F}$$

We have now two condition equations to be satisfied.

$$N \gtreqless \left(1+\frac{1+\cos 2\varkappa}{K}\right)(1+F^2(x)) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and

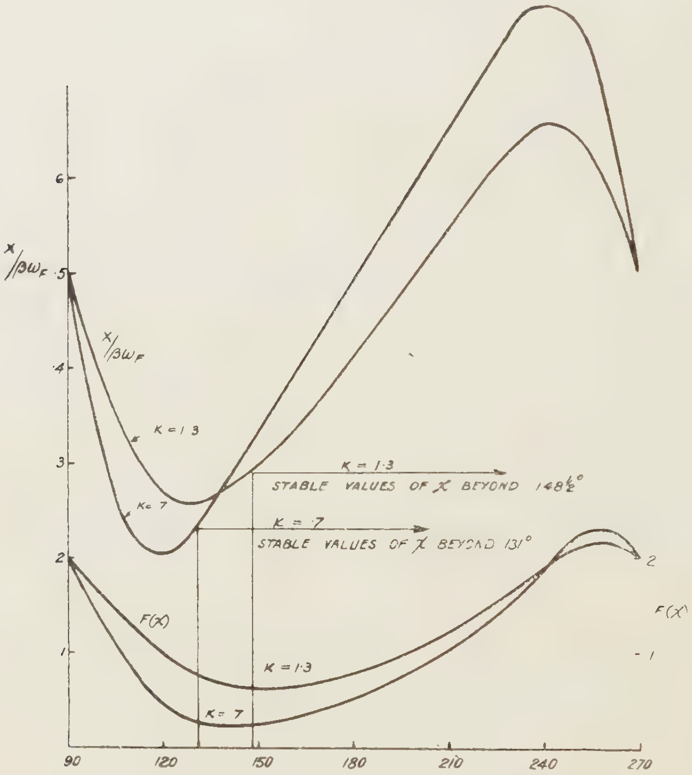
$$N \gtreqless 2+(\tan\gamma_{1F}-\cot\gamma_{1F})\,F(x)$$

and from these

$$\tan\varkappa = \frac{2}{-K(\tan\gamma_{1F}+\cot\gamma_{1F})} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

This case is sufficiently discussed. We know that in order to maintain the motor in a stable state

$$\varkappa > \tan^{-1}\frac{2}{-K(\tan\gamma_{1F}+\cot\gamma_{1F})}$$



Graph II.—Motor and Fork (Untuned, without Reaction).

and substituting values of $\varkappa$ between this and $3\pi/2$ in the condition equation we can determine values of N, and so of $\frac{X}{\beta\omega_F}=A_1{}^2/\beta L_1\omega_F$ to maintain this condition.

Again, we can calculate the change in $1-\omega^2/\omega_F{}^2$ as $\varkappa$ varies in value between the above limits.

Further, from the relation $i_1\cos\varkappa=\frac{2P}{V}$, i_1 is determined, and the amplitude of the fork is determinate from the relation

$$(\alpha D^2+\beta D+\gamma)y=A_1 i_1$$

This decides the dimensions of the fork, since the amplitude of vibration must be kept within reasonable limits.

In Graph II, $X/\beta\omega_F$ is plotted against values of $\varkappa$ between $\pi/2$ and $3\pi/2$, and the point beyond which the system is stable when

$$\varkappa > \tan^{-1}\frac{2}{-K(\tan\gamma_{1_F}+\cot\gamma_{1_F})}$$

is indicated. Notice that at this point $X/\beta\omega_F$ is increasing in value.

For this graph K is taken as 0·7 and 1·3, and the minimum value of $X/\beta\omega_F=0{\cdot}295$. Guess the resistance r_1 of the motor and the resistance R_1 of the fork, small compared with $1/s_1$ and $\frac{L_1 s_1 \omega_F}{1+r_1+R_1 s_1}=1$. The minimum value of $X/\beta\omega_F$ is $\frac{2{\cdot}95}{m-1}$ if $K=1{\cdot}3$ and $A_2=A_1$, but a very much greater value of $X/\beta\omega_F$ will be required to give a stable system, since it increases in value at this point.

Compare this figure with $X/\beta\omega_F=\frac{2}{m-1}$ for the fork alone, given in the author's Paper (quoted).

Case (2).—With Grid Reaction from the Motor.

Grid conductance negligible and reaction from the grid winding of the motor.

In our treatment of the motor alone we decided that for a non-conducting grid we would make $\mu=\sqrt{2}$, $\tan\gamma=1$ and $\theta=3\pi/4$.

Substituting these values in the frequency and condition equations

$$1-\frac{\omega^2}{{\omega_F}^2}=\frac{\beta\ [}{\alpha\omega_F}\,\frac{K\tan\gamma_{1_F}-1+\overline{\cos 2\varkappa}}{K+\sin 2\varkappa} \quad \dots\dots\dots \quad (15)$$

and

$$N \geqq \frac{(K\tan\gamma_{1_F}-\overline{1+\cos 2\varkappa})^2+(K+\sin 2\varkappa)^2}{K(K+\sin 2\varkappa)} \quad \dots\dots \quad (16)$$

In this case the anode E.M.F. which is required to lag behind the back E.M.F. of the motor by an angle greater than $\pi-\gamma$ is

$$\frac{NV_M}{F(x)-\tan\gamma_{1_F}+(1-N+\tan\gamma_{1_F}F(x))\,\frac{D}{\omega}}$$

so that $\frac{1-N+\tan\gamma_{1_F}F(x)}{\tan\gamma_{1_F}-F(x)}$ requires to be $<\tan\gamma_{1_F}$—i.e.,

$$N \geqq 1-\tan^2\gamma_{1_F}+2\tan\gamma_{1_F}F(x)$$

Combining this with

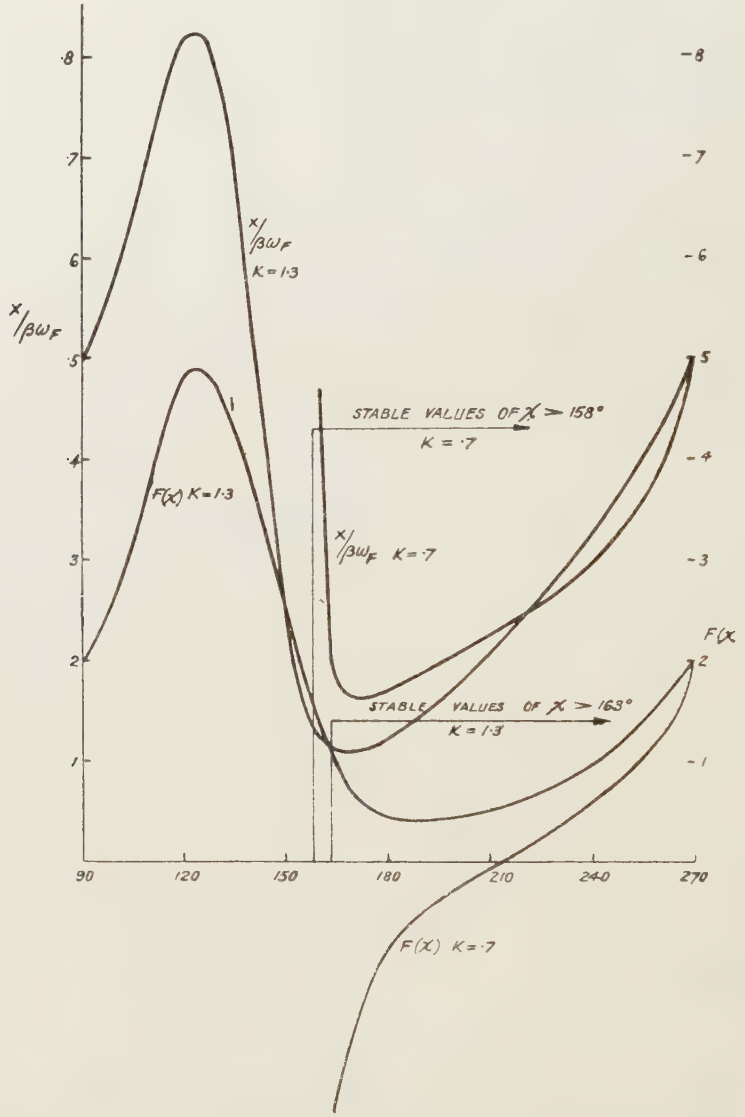
$$N \geqq \left(1+\frac{\sin 2\varkappa}{K}\right)(1+F^2(x)) \text{ above} \quad \dots\dots\dots \quad (17)$$

$$\varkappa \geqq \tan^{-1}\frac{2}{-K(1+\tan^2\gamma_{1_F})} \quad \dots\dots\dots\dots \quad (18)$$

In the Graph III, $X/\beta\omega_F$ is plotted against values of $\varkappa$ between $\pi/2$ and $3/2\pi$, and the point beyond which the system is stable when $\varkappa>\tan^{-1}\frac{2}{-K(1+\tan^2\gamma_{1_F})}$ is shown.

K for this case is taken as 1·3 and 0·7, $\tan\gamma_{1_F}=2$, and $m=11$. The minimum value of $X/\beta\omega_F=0{\cdot}1185$ when $K=1{\cdot}3$ and $A_2=A_1$.

Notice that now at this point $X/\beta\omega_F$ is increasing very slowly in value, so that for a value, say, 2, the system is stable over quite a large range of $\varkappa$; also that values of $F(x)$ reach lower figures than in the case without reaction.



Graph III.—Motor and Fork (Untuned, with Reaction).

Similarly to the preceding case, values of $\varkappa < 163^\circ$ can be put into the frequency and condition equations, and the fork dimensions can be determined from the necessary value of i to maintain the load.

MOTOR WINDINGS IN SERIES WITH CONDENSER TUNED FORK WINDINGS (FIG. 2).

If this circuit is systematically analysed the circuit equation resulting is

$$\left[\begin{aligned}&(G_1H_1+s_1\overline{R_1+L_1D})(G_2H_2+s_2\overline{R_2+L_2D})\left(1+\frac{D^2}{\omega_F{}^2}+\frac{\beta}{a\omega_F}\cdot\frac{D}{\omega_F}\right)\\&+(G_2H_2+s_2\overline{R_2+L_2D})(s_1+c_1DG_1)\frac{A_1{}^2}{a\omega_F{}^2}D\\&+(G_1H_1+s_1\overline{R_1+L_1D})(s_2+c_2DG_2)\frac{A_2{}^2}{a\omega_F{}^2}D-\frac{ms_1A_1A_2}{a\omega_F{}^2}D\end{aligned}\right]i_1=$$

$$\left[\begin{aligned}&(G_2H_2+s_2\overline{R_2+L_2D})\left\{H_1\left(1+\frac{D^2}{\omega_F{}^2}+\frac{\beta}{a\omega_F}\,\frac{D}{\omega_F}\right)+\frac{A_1{}^2}{a\omega_F{}^2}c_1D^2\right\}\\&\qquad+H_1(s_2+c_2DG_2)\frac{A_2{}^2}{a\omega_F{}^2}D\end{aligned}\right]s_1V_M+$$

$$\left[\begin{aligned}&m\overline{1+r_2s_2}\left\{H_1H_2\left(1+\frac{D^2}{\omega_F{}^2}+\frac{\beta}{a\omega_F}\cdot\frac{D}{\omega_F}\right)+\frac{A_1{}^2c_1D^2H_2+A_2{}^2c_2D^2H_1}{a\omega_F{}^2}\right\}\\&\qquad+\frac{A_1A_2Ds_2}{a\omega_F{}^2}\end{aligned}\right]s_1V_A \quad . \quad . \quad . \quad (19)$$

where

$$G_1=1+r_1s_1+l_1s_1D \qquad H_1=1+R_1c_1D+L_1c_1D^2$$
$$G_2=1+r_2s_2+l_2s_2D \qquad H_2=1+R_2c_2D+L_2c_2D^2$$

TUNED FORK, WITHOUT MOTOR.

When we deal with the *Tuned Fork circuit alone*—i.e., when there are no motor windings in series with the fork windings in the electrode circuits, and we tune condensers so that

$$\frac{1+R_1s_1}{L_1c_1}=\frac{1+R_2s_2}{L_2c_2}=\omega^2{}_F$$

The frequency equation becomes (neglecting small terms if $\beta/a\omega_F$ is small):—

$$1-\omega^2/\omega_F{}^2=-\beta/a\omega_F\left[\frac{X_1}{\beta\omega_F}\ \frac{L_1s_1}{R_1c_1+L_1s_1}+\frac{X_2}{\beta\omega_F}\ \frac{L_2s_2}{R_2c_2+L_2s_2}\right]$$

where $A_1{}^2=X_1L_1$ and $A_2{}^2=X_2L_2$.

Notice that if $X/\beta\omega_F$ is small—and it can be very small with tuned systems—the difference between the fork note and the note of the maintained system can be very much smaller than these differences in untuned systems given in the author's Paper quoted.

The maintained note is slightly higher than the true fork note, and the plate and grid windings require to be oppositely wound.

If $A_2=-A_1$, $X_2=X_1=X$, and $R_2=R_1=R$, the condition equation becomes

$$X/\beta\omega_F\geqq\frac{\dfrac{R}{L\omega_F}\left(1+\dfrac{Ls_1}{Rc_1}\right)\left(1+\dfrac{Ls_2}{Rc_2}\right)}{\dfrac{m}{1+Rs_2}\,\dfrac{Ls_1}{Rc_1}-\left(2+\dfrac{Ls_1}{Rc_1}+\dfrac{Ls}{Rc_2}\right)} \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Guess $\frac{s_2}{c_2}=0{\cdot}2\frac{s_1}{c_1}$, $\frac{R}{L\omega_F}=0{\cdot}3$ and $m=7{\cdot}2$, the optimum value of $s_1/c\omega_F$ is about 0·8 and

$$X/\beta\omega_F=0{\cdot}1285.$$

If the grid conductance is negligible

$$1-\omega^2/\omega_F{}^2=-\beta/a\omega_F\frac{X/\beta\omega_F}{1+\frac{Rc_1}{Ls_1}}$$

and

$$X/\beta\omega_F\geqq\frac{Rc\omega_F\left(1+\frac{Rc_1}{Ls_1}\right)}{m-\left(1+2\frac{Rc_1}{Ls_1}\right)}$$

Compare this with the case in which we tune the condensers so that $Lc_1\omega^2{}_F=Lc_2\omega^2{}_F=1$. In this case

$$1-\frac{\omega^2}{\omega_F{}^2}=-\beta/a\omega_F\frac{(Rc\omega_F+2X/\beta\omega_F)}{1+\frac{Rc}{Ls_1}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (22)$$

and

$$X/\beta\omega_F\geqq\frac{Rc\omega_F\left[\left(1+\frac{Rc}{Ls_1}\right)^2+R^2c^2\omega_F{}^2\right]}{\overline{m+1}\left(1+\frac{Rc}{Ls_1}\right)-2\left\{\left(1+\frac{Rc}{Ls_1}\right)^2+R^2c^2\omega_F{}^2\right\}} \quad . \; . \; . \; . \quad (23)$$

Tuned in this way, the difference between the note of the system and the true fork note is greater, and the value of $X/\beta\omega_F$ is greater also. However, the case is of interest because I have dealt with the motor problem with the circuits tuned in this way, and the solutions can thus be compared.

There are other modes in which the system is maintained in vibration with the coils similarly wound, which depend to a much greater extent on the tuning of the condensers ; but the solution given above is essentially that of a *fork oscillator* as opposed to a *circuit oscillator*, and requires the magnet coils *oppositely* wound.

With the coils *similarly* wound, for example, circuit notes may be maintained independently of any vibration of the fork, since the coupling is in the right direction to maintain such notes ; whereas with the coils oppositely wound the only circuit note that can possibly be supported is a higher frequency difference note, and the coupling cannot easily be made sufficiently great to sustain this mode.

The principles outlined in this case as well as those of the Fork Papers mentioned are not limited to valve maintained forks, but must apply to self-maintained vibrating systems generally, such as the Campbell Hummer and various reed devices using carbon microphones to relay power. They do not exclude even contact driven devices if the phase of the make and break of the current supply is determinable.

TUNED FORK WITH MOTOR.

Including the motor in the problem the system is better studied when divided up into particular cases.

By way of simplifying expressions and dealing with the cases in which the grid conductance is negligible

R_1 is made $= R_2 = R$
L_1 ,, $= L_2 = L$
c_1 ,, $= c_2 = c$
A_1 ,, $= A_2 = A$
r_1 ,, $= r_2 = r$
l_1 ,, $= l_2 = l$

$\dfrac{s_1}{1+r_1 s_1}$ is written s, $s_2=0$, and the condensers are adjusted so that $1-Lc\omega_F{}^2 = \delta_1$—a small quantity.

Substituting

$$-\cos\varkappa\left(\cos\varkappa+\sin\varkappa\frac{D}{\omega}\right)\frac{\omega}{\omega_F}\frac{2}{K}i_1=V_M s$$

and
$$-\cos\varkappa\left(\cos\overline{\varkappa-\theta}+\sin\overline{\varkappa-\theta}\frac{D}{\omega}\right)\frac{\mu}{m}\frac{\omega}{\omega_F}\frac{2}{K}i_1=V_A s$$

in the circuit equation, this equation can be split up into :—

Frequency Equation

$$1-\frac{\omega^2}{\omega_F{}^2}=-\frac{\beta}{\alpha\omega_F}\frac{[(Rc\omega_F+2\overline{1-\delta}X/\beta\omega_F)Y_x+\delta Z_x]}{Rc\omega_F Z_x-\delta Y_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

and Condition Equation

$$X/\beta\omega_F \geqq \frac{(R^2c^2\omega_F{}^2+\delta^2)(Y^2{}_x+Z^2{}_x)}{\overline{m+1}Ls\omega_F(Rc\omega_F Z_x-\delta Y_x)-2\overline{1-\delta}Rc\omega_F(Y^2{}_x+Z^2{}_x)} \quad . \quad (25)$$

for the solution we are concerned with where

$$Y_x=Rs\,(1-lc\omega_F{}^2)+\delta+\frac{2\cos\varkappa}{K}[\delta(\cos\varkappa+\mu\cos\overline{\varkappa-\theta})-Rc\omega_F(\sin\varkappa+\mu\sin\overline{\varkappa-\theta})]$$

$$Z_x=\overline{L+\delta l}\,.\,s\omega_F+Rc\omega_F+\frac{2\cos\varkappa}{K}[\delta(\sin\varkappa+\mu\sin\overline{\varkappa-\theta})+Rc\omega_F(\cos\varkappa+\mu\cos\overline{\varkappa-\theta})]$$

Case I.

When there is no reaction by the motor in the grid circuit, V_A, r_2 and l_2 have no value, and the anode circuit equation can be re-written

$$\left[1/s+lD+\frac{R+LD}{\delta+RcD}\right]i_1$$
$$=\left[1-\frac{\overline{m+1}X/\beta\omega_F LD}{\left(F(\varkappa)+\dfrac{D}{\omega_F}\right)\left\{(1/s+lD)(\delta+RcD)^2+(R+LD)(\delta+RcD)\right\}+ +X/\beta\omega_F LD\left\{m-1+2\left(1-lc\omega^2+\dfrac{cD}{s}\right)(\delta+RcD)\right\}}\right]V_M \quad . \quad (26)$$

where $1-\frac{\omega^2}{\omega_F{}^2}=\frac{\beta}{\alpha\omega_F}F(\varkappa)$, and where the last term represents the portion of the anode E.M.F. for which the fork is responsible, and is intended to maintain steady in the case of a sudden load on the machine.

Clearly, if there is to be any lag of the current i_1 behind the resultant E.M.F. in the anode circuit, the inductance l of the motor winding must be greater than the inductance L of the fork anode winding, or δ must have positive value.

In order to provide stable running conditions this lag must have some value, and the power of the motor is a maximum when its value is $\pi/2$. This proportion of power taken by the motor is increased in a particular system by increasing the value of δ as well as by increasing the ratio l/L.

The condition that the anode E.M.F. for which the fork is responsible must lag behind the E.M.F. of the motor by an angle greater than π, less the lag of the current behind the resultant E.M.F. in the anode circuit, gives another relation—i.e.,

$$\begin{aligned}\overline{m+1}X/\beta\omega_F Ls\omega_F(\delta Z-Rc\omega_F Y) &\geqq \\ [(\delta^2+R^2c^2\omega_F{}^2)F(\varkappa)-\delta 2\overline{1-\delta}X/\beta\omega_F](Y^2+Z^2)& \\ -[\delta^2+R^2c^2\omega_F{}^2+Rc\omega_F 2\overline{1-\delta}X/\beta\omega_F]2YZ& \quad . \quad . \quad . \quad . \quad (27)\end{aligned}$$

where $Y=Rs(1-lc\omega_F{}^2)+\delta$ and $Z=\overline{L+\delta l}\,.\,s\omega_F+Rc\omega_F$.

This condition equation compared with the condition equation given earlier gives the minimum value of $\varkappa$ for which the system is stable—i.e.,

$$\varkappa_{\text{min.}}=\tan^{-1}\frac{2(Rc\omega_F Y-\delta Z)}{K(Y^2+Z^2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

If the equations (24) and (25) are examined it will be appreciated that if $R/L\omega_F$ is small and Ls/Rc is greater than 2·5, the optimum value of δ is very nearly zero.

If Ls/Rc is small, negative values of δ will improve the value of both $X\beta/\omega_F$ and $F(\varkappa)$; but there is no point in making Ls/Rc small, and the examples shown in Graph IV are for a case in which $Lc\omega_F{}^2=1$—i.e., $\delta=0$, $Rc\omega_F=0{\cdot}2$, $Rs=0{\cdot}1$, $Ls\omega_F=0{\cdot}5$, $l/L=3$ and $m=11$.

$X/\beta\omega_F$ and $F(\varkappa)$ are shown for values of $K=0{\cdot}7$ and $1{\cdot}3$.

Case II.

When there is reaction by the motor in the grid circuit as well as in the anode circuit, say that

$$V_A=\frac{\frac{\mu}{m}V_M}{\cos\theta+\sin\theta\frac{D}{\omega}}$$

$$=-\frac{\mu}{m}\cos\varkappa\left(\cos\overline{\varkappa-\theta}+\sin\overline{\varkappa-\theta}\frac{D}{\omega}\right)\frac{\omega}{\omega_F}\frac{V^2}{2P}i_1$$

The circuit now becomes

$$\left[1/s+lD+\frac{R+LD}{\delta+RcD}\right]i_1=$$

$$\left.\frac{\overline{m+1}\,X/\beta\omega_F\,LD}{\left\{\frac{2X}{\beta\omega_F}LcD^2+(\delta+RcD)\left(F(x)+\frac{D}{\omega_F}\right)\right\}\left\{R+LD+(\delta+RcD)\,(1/s+lD)\right\}+\overline{m+1}\frac{X}{\beta\omega_F}\,LD}\right]\left(1+\frac{\mu}{\cos\theta+\sin\theta\frac{D}{\omega}}\right)V_M \qquad (29)$$

and what has already been said about the necessity for the current i to lag behind the resultant E.M.F. in the anode circuit for Case I applies equally in this case—that is, l must be greater than L or δ must have a positive value. The portion of the anode E.M.F. for which the fork is responsible and is intended to maintain steady for a period, in case of a sudden temporary load on the machine, is equal to the similar portion of the anode E.M.F. of Case I, multiplied by the operator.

$$\left(1+\frac{\mu}{\cos\theta+\sin\theta\frac{D}{\omega}}\right)$$

The effect of introducing reaction by the motor in the grid circuit of the valve is to increase the lead of the back E.M.F. of the motor on the fork portion of the anode E.M.F. by an amount

$$\tan^{-1}\frac{\mu\sin\theta}{1+\mu\cos\theta}$$

The condition that the anode E.M.F. for which the fork is responsible must lag behind the E.M.F. of the motor by an angle greater than π, less the lag of the anode current behind the resultant E.M.F. in the anode circuit gives the relation

$$\begin{aligned}\overline{m+1}Ls\omega_F\,(\delta Y+Rc\omega_F\,Z)X/\beta\omega_F \gtreqless\\ [\delta^2+Rc\omega_F(Rc\omega_F+2\overline{1-\delta}X/\beta\omega_F)](Z^2-Y^2)-\\ [(\delta^2+R^2c^2\omega^2{}_F)F(x)-\delta 2\overline{1-\delta}X/\beta\omega_F]2YZ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)\end{aligned}$$

and the minimum value of $\varkappa$ for stable running conditions is

$$\tan^{-1}\frac{0{\cdot}2(Rc\omega_F Z+\delta Y)}{-K(Y^2+Z^2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Two examples of this case are shown in Graphs IV and V. The values of μ and θ selected are those decided upon when we considered the motor and valve without the fork, i.e., $\mu=2\cos\gamma$ and $\theta=\pi-\gamma$.

$Lc\omega_F{}^2=1$, $l/L=3$, $Rs=0{\cdot}1$, $Rc\omega_F=0{\cdot}2$, $Ls\omega_F=0{\cdot}5$ and $m=11$. $X/\beta\omega_F$ and $F(\varkappa)$ are shown for values of $K=0{\cdot}7$ and $1{\cdot}3$.

Notice that in this case the useful portion of the $F(\varkappa)$ curve is negative—i.e., the note of the maintained system is slightly higher than the true fork note—also that the values of $X/\beta\omega_F$ for stable running are less than any of the earlier cases, and compare with the tuned fork case without the motor.

For the tuned systems the amplitude of the fork is determined from the relation

$$\left[\delta F(x) - Rc\omega_F - 2\overline{1-\delta}X/\beta\omega_F + (Rc\omega_F F(x)+\delta)\frac{D}{\omega}\right]y = -\frac{A}{\beta\omega_F}i_1 \quad . \quad . \quad . \quad (32)$$

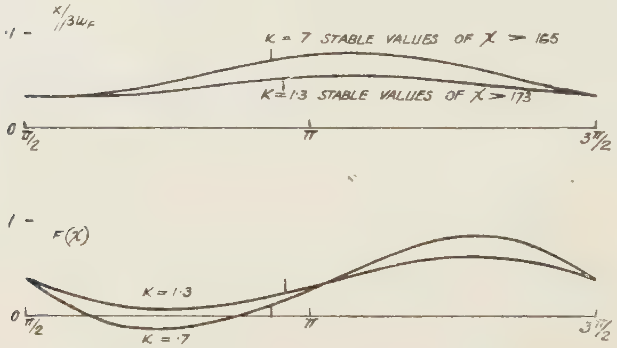
and since i is $-\frac{2P}{V\cos\varkappa}$ the design conditions are as estimable as they are for the untuned cases.

As an example, say

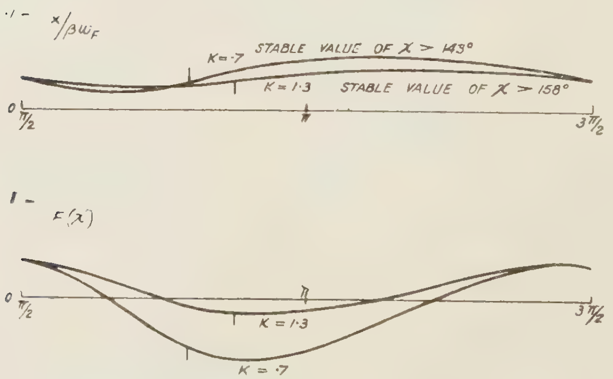
$P=0{\cdot}045$ watts. $V=50$ volts.
$\varkappa=195°$. $\delta=0$. $Rs=0{\cdot}1$. $Rc\omega_F=0{\cdot}2$.
$\omega_F=2{,}000\pi$. $F(x)=-0{\cdot}2$. $X/\beta\omega_F=0{\cdot}05$.

$$y=\frac{-\frac{A_1}{\beta\omega_F}i_1}{\sqrt{(\delta F(x) - Rc\omega_F - 2\overline{1-\delta}X/\beta\omega_F)^2 + (Rc\omega_F F(x)+\delta)^2}}$$

Guess $\beta=20$ and $L=2\times10^9$, $\frac{A}{\beta\omega_F}=\sqrt{X/\beta\omega_F \, . \, L/\beta\omega_F}$ and $y=0{\cdot}174m/m$.



Graph IV.—Motor and Fork. Condenser Tuned $1-LC\omega^2=0$: Without Reaction on Grid.



Graph V.—Motor and Fork. Condenser Tuned $1-LC\omega^2=0$: With Reaction on Grid.

The results given above indicate the influence of the phase angle $\varkappa$ or the lag of the anode current behind the back E.M.F. of the motor in the anode circuit, on the maintained note of the system and the value of driving term $X/\beta\omega_F$, and so determine the design conditions.

All of these results depend entirely, for the approximation to truth, upon the value of $\beta/\alpha\omega_F$ being a very small quantity, and of the possibility of being able to ignore all but fundamental frequencies. The grid conductance has been ignored in the cases which include the motor, for the reason that the expressions are sufficiently cumbersome without it; however, consideration of the problem on the fork alone, in which it has been included, will give a clue to its effect in the motor problem.

In the foregoing work an effort has been made to determine design conditions, and to evaluate the differences between maintained notes of the systems and true fork notes for various combinations of fork systems with synchronous alternating current motors.

Both untuned and tuned systems are dealt with (as well as a special case dealing with a tuned fork without a motor), and the effect of introducing reaction from the motor in the grid circuit of the valve has been included.

Various other arrangements for tapping energy from the anode circuit may be made, and other arrangements for introducing reaction from the motor in the grid circuit are possible ; but for similar powers and similar energy distribution the results must be similar to those given above.

In practice the design of efficient systems without reaction from the motor on the grid is difficult, and it is useful in any case to have a method of running the machine up to synchronism with the fork.

Where larger powers are concerned, or where cost of material and efficiency are not considered, it is expedient to amplify an E.M.F. from the anode circuit through succeeding stages of a power amplifier to the motor, and in this case the difference between the maintained note and the true fork note can be made independent of $\varkappa$ entirely, although in the examples shown the variation in $\varkappa$ is so small, in useful cases, that the maintained note is practically constant.

I wish to express my indebtedness to Mr. A. Hinchliffe, who has made up material and has collected a large amount of experimental evidence from the work ; also to the Director of Artillery for his permission to offer the Paper for publication.

DISCUSSION.

Prof. O. W. RICHARDSON congratulated the author on the thoroughness of his investigation and on the precision attained by means of his design. When he saw a monumental Paper like the present devoted to engineering problems arising out of the application of thermionic systems, his feelings resembled those of a hen which has laid an egg and hatched an elephant.

Mr. R. S. WHIPPLE said that the instrument was a chronograph of high accuracy, but the actual error must depend ultimately on the accuracy of the fork. Presumably for an error of 1 part in 10,000 an "Elinvar" fork must be used, or a fork kept at constant temperature. He congratulated the author not only on the thoroughness of his theory, but on the excellence of his mechanical design ; his drawings had been such that they could be used as shop drawings with very little preparation.

Dr. D. OWEN enquired whether the author had measured the mechanical amplitude of the fork directly as distinct from the anode current ? In a case like the present, where the fork is coupled to an otherwise independent system, the amplitude was not independent of the frequency, and it would be interesting to learn what was the functional relation between frequency and amplitude.

AUTHOR'S reply : I wish to thank Professor Richardson for his eulogy of the work and at the same time express contrition for having contributed to the Presidential hen's little accident.

The reason for the design was not so much to measure time accurately, as to draw an accurate scale in which the finer sub-divisions of time occupy their proper places, and are uniform parts of the unit. For this purpose the ordinary steel forks supplied by the Cambridge Instrument Co., have been sufficiently accurate time keepers, but I have contemplated the use of an "Elinvar" fork and hope to secure one eventually. The fact that the small motors work with a power consumption, electrical and mechanical of a few milliwatts only, is proof of the excellence of the Cambridge Company's workmanship in their construction.

Dr. D. Owen asks a very important question. The frequency equation in the Paper for motors coupled to condenser tuned forks, gives a solution depending on the phase lag of the anode current behind the back electro-motive force of the motor, and if the machine hunts the frequency varies to an extent I have endeavoured to predetermine. With the motors we have made, and the small powers we deal with, although phase variation would be measurable, the accurate measurement of phase angles would be difficult. On the other hand, we could measure the variation of frequency with amplitude, varying the anode supply voltage, and although such a measurement would leave a number of quantities indeterminate, it would be of practical value.

We always keep the amplitude of the fork small, and calibrate and use the instrument under identical running conditions.

I have made isolated measurements of amplitude with 1,000 cycle and 50 cycle valve maintained forks, and have found the frequency variation of the latter (a fork without condenser tuning) with amplitude greater than I expected; however, these measurements were associated with varying methods of control, and are not worth quoting in this connection.

The damping coefficient, 20 dyne c.m. seconds, given as a numerical example in the Paper for a 1,000 cycle fork is small for forks I have had experience with, and values three times this figure are nearer the mark.

XVI.—MEASUREMENTS OF ABSORPTION COEFFICIENTS OF LIGHT FILTERS.

By G. M. B. DOBSON, *D.Sc.*, and I. O. GRIFFITH, *M.A.*, Clarendon Laboratory, Oxford.

Received January 3, 1927.

ABSTRACT.

A new spectro-photographic method of measuring absorption coefficients is described. A portion of the slit of a spectrograph is covered by the absorbing medium, and in front of the photographic plate or of the slit, a neutral wedge is placed. The resulting spectogram consists of two parts, one due to light which has passed through the filter and the wedge, the other to light which has traversed the wedge only. It is shown how, from a knowledge of the distance between two points, one in each part of the spectogram, which are of the same density, the absorption coefficient of the filter at any wavelength may be determined. The source of light need not be constant.

WE have had occasion recently to measure the absorption coefficients of various filters for radiation, both in the visible and ultra-violet parts of the spectrum, and in the course of our work we have devised a simple and accurate method of determining these coefficients.

Ordinarily the intensity of a beam of light is measured by a spectro-photometer fitted with a photo-electric cell or some other type of energy measuring instrument, and the absorption of the filter under test is balanced at various wavelengths against that of a neutral optical wedge or some other standard of absorption. In order to allow for variations in the source of light, the beam is split into two parts, one beam traversing the filter under test, the other passing through the standard absorbing substance. Where this is not possible, the substance under test and the absorption standard are brought alternately into the path of the beam.*

This type of apparatus is necessarily elaborate, and, moreover, it is difficult to obtain a constant source of light and maintain it over an extended period, especially when working in the ultra-violet portion of the spectrum.

In the method which we have adopted, although only one beam of light is used, the constancy of the light source is not essential, and the exact time of exposure is immaterial, provided that sufficient light is transmitted.

The apparatus consists essentially of an ordinary spectograph with its accessories, and a neutral wedge whose density-gradient is known. The greater part of the slit of the spectograph is covered by the filter whose absorption coefficient is to be measured. Also the whole slit, or if more convenient the photographic plate, is covered by the neutral wedge. The resulting spectogram consists of two parts, in the one the light has suffered absorption by the wedge and filter, in the other the wedge alone has been operative. For any given wavelength there will be pairs of points, one in each part of the spectogram, at which the photographic density is the same. It will be shown later that the distance between any such pair of points, multiplied by the density-gradient of the wedge, gives the absorption coefficient of the filter for the wavelength under consideration.

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* Thorne Baker, Journ. Sci. Inst., p. 13, October (1925). W. D. Haigh, Journ. Sci. Inst., p. 211, April (1926).

DESCRIPTION.

In Fig. 1, *AB* represents the slit of the spectograph, *WW* a neutral wedge, *FF* the filter whose absorption coefficient is to be measured.

The wedge has a reference line at its thick end, and the filter is placed so that its edge comes to within about 2 mm. of the reference line at *O*. A spectrogram taken through this combination will appear somewhat as shown on the left of Fig. 1, where the source of light is a carbon arc and *FF* a blue filter. The lower part *YY′* corresponds to light which has passed through both wedge and filter. In the upper part *XX′* the image depends solely on the absorption by the wedge.

When the filter has a wide opaque strip at its edges, as is the case with liquids in glass cells, a rhomboid prism may be placed in front of the filter as shown in Fig. 2.

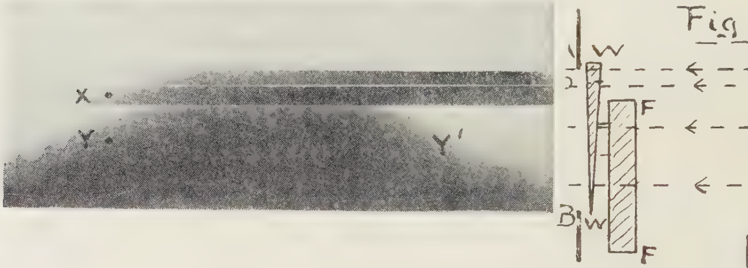
Fig 1

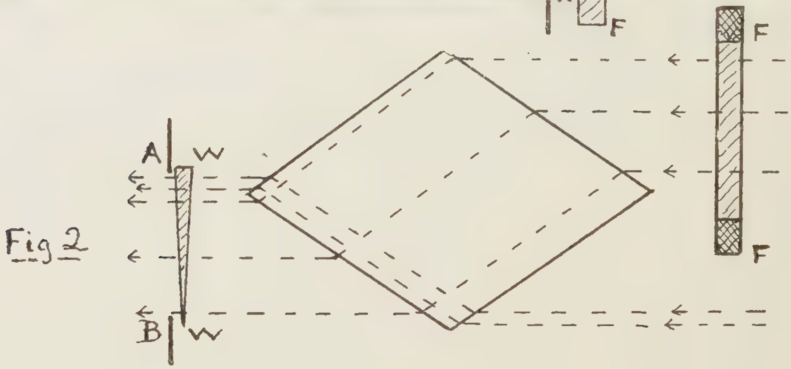
Fig 2

THEORY.

If I_0 is the intensity of the light incident on the instrument, and if x is the distance of a point X in the portion XX' from the image of the thin end of the wedge, the intensity of the light falling on the photographic plate at X is given by

$$I_x = I_0 \, . \, 10^{-Kx}$$

K being the known wedge constant.

At a point Y in the lower part of the diagram, whose distance from the same reference line is y, the light intensity is I_γ where

$$I_y = I_0 \, . \, 10^{-\mu\lambda} \, . \, 10^{-Ky}$$

μ_λ being the absorption coefficient of the filter for the wavelength λ.

If the densities at X and Y are the same

$$I_x = I_y$$

or

$$I_0 \, . \, 10^{-Kx} = I_0 \, . \, 10^{-Ky \cdot \mu_\lambda}$$

or

$$\mu_\lambda = K(y - x)$$

$$= Kz \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where z is the distance between X and Y.

It will be noticed that no characteristic of the plate or its treatment enters into this equation. We make the one assumption that if the sensitivity and the development are uniform over a small portion of the plate then for any given wavelength equal intensities give equal densities.

METHOD.

The plate after exposure and development is placed on the stage of any good micro-photometer, and the density is noted at a point X, say 1 mm. below the reference line on the part of the image where light has not passed through the filter. Another point Y is found on the lower part of the image, at the same wavelength, and at which the density is the same as at X. The measurements may be repeated, using slightly different positions of X (all at the same wavelength) and correspondingly different positions of Y. The mean value of the values obtained for the distance between X and Y is substituted in equation (1), giving the required absorption coefficient.

It is of course necessary that the incident radiation should be of uniform intensity over the slit. In practice the radiation does not fall on the slit or the wedge as a parallel pencil. It is therefore desirable to calibrate the wedge *in situ*, and not to take values of K measured in parallel light.

It is obvious that any variation in the intensity of the source has no effect on the relative densities at X and Y. Line spectra, using a wide slit, can of course be employed instead of continuous spectra. The wedge or filter may be placed either in front of the slit or of the photographic plate.

By taking the precautions for uniform illumination of the slit and uniform development of the plate which we have described elsewhere, results can easily and readily be obtained with an accuracy of about 1 per cent.

NOTE ADDED MARCH 10TH, 1927.

Since the above was written we have slightly modified the arrangement in order to reduce the exposure necessary. The filter which is being measured is made to cover a smaller part of the slit than indicated above. The point X in the above diagram is then taken near the densest edge of the image where the light has passed through both wedge and filter, and a point Y of equal density is found on the upper part of the image where the light passes through the wedge only. The theory is identical with that given above, but since the thinner parts of the wedge are used, the exposures can be reduced.

Stray Light.—Where a very strongly absorbing filter is being measured errors are likely to arise from light scattered from a part of the spectrum which is freely transmitted, causing fogging over other parts of the spectrum, during the long exposures which are necessary to get a measurable image at the wavelengths which are strongly absorbed. This can be eliminated by using subsidiary colour filters covering the *whole* slit which reduce the light of wavelength not required. To get reliable measures in all parts of the spectrum it may be necessary to take two or three spectrograms with suitable exposures and suitable additional filters covering the whole slit; this, however, entails very little extra work.

It is also possible that as the light from the upper parts of the spectrum is very intense compared with the lower, scattered light from the former may fog the latter. When this is the case the points X and Y will necessarily be a long distance apart, and the major part of the light causing the scattering can be eliminated by raising the filter being measured until its edge approaches the point Y in the upper part of the image. Again, it may be necessary to take two or three spectra, a part of the spectrum being measured on each.

XVII.—A COMPARISON OF THE BEHAVIOUR IN THERMAL DIFFUSION OF NITROGEN AND CARBON MONOXIDE, AND OF NITROUS OXIDE AND CARBON DIOXIDE.

By T. L. Ibbs, *M.C.*, *Ph.D.*, Lecturer in Physics, and L. Underwood, *M.Sc.*,
University Research Scholar, University of Birmingham.

Received December 17, 1926.

(*Communicated by* Prof. S. W. J. Smith, *F.R.S.*)

Abstract.

The thermal separation obtained in gas mixtures containing nitrogen is compared with that obtained in mixtures containing carbon monoxide. A similar comparison is made of mixtures containing carbon dioxide or nitrous oxide. The gas analysis required in the measurement of the effect is made by means of the Shakespear katharometer. The behaviour of nitrogen is found to be similar to that of carbon monoxide. The effect given by carbon dioxide is generally a little greater than that given by nitrous oxide.

The pairs of gases examined provide a special case for the application of the Enskog-Chapman theory, as in each pair the molecular weights and mean collision areas are the same. It can thus be deduced that the molecular field of nitrogen is similar to that of carbon monoxide, and that the field of carbon dioxide differs little from that of nitrous oxide.

THE Enskog-Chapman theory* of thermal diffusion has shown how this effect in a mixture of two gases will depend on the masses and diameters of the gas molecules, and on the proportions of the gases in the mixture. The effect has also been shown to depend on the nature of the molecules: it will be greatest for molecules which behave like rigid elastic spheres, and will disappear entirely for Maxwellian molecules obeying an inverse fifth power law. A comparison of the observed effect and the maximum calculated effect will give information on the nature of the molecular fields of force. The theory has already been confirmed experimentally† in a number of important points.

The present series of experiments originated in a suggestion made by Prof. S. W. J. Smith as to the possibility of investigating whether any thermal separation can be obtained in a mixture of gases such as nitrogen and carbon monoxide, where the molecular weights and sizes are practically the same. For this special case the theory predicts that the effect will disappear, whatever may be the nature of the fields of force. As the thermal conductivities of nitrogen and carbon monoxide are also nearly the same, this pair does not lend itself to direct examination by the method employed by one of the present writers in previous experiments, in which the Shakespear katharometer was used for gas analysis. Any small amount of separation which might be obtained would be difficult to measure in this case. The suggestion, however, led to the idea of mixing the gases separately with a suitable subsidiary gas of considerably different thermal conductivity and comparing the effects which can be observed. For example, using hydrogen as a subsidiary gas, the effect in a mixture of carbon monoxide and hydrogen can be compared with that in a mixture of nitrogen and hydrogen. As the gases forming the two mixtures are practically the same in molecular size and weight, any difference in the thermal diffusion effect observed will be due to a difference in the forces arising during collision. If the effect is the same for the two pairs, it will indicate a similarity of the molecular fields of force of the two gases carbon monoxide and nitrogen. A similar system of comparison can be applied to carbon dioxide and nitrous oxide, as these two gases have also very nearly the same molecular weight and size.

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* Enskog, D., Phys. Zeit., Vol. 12, p. 538 (1911); Ann. d. Phys. (4), Vol. 38, pp. 742, 750 (1912); Chapman, S., Proc. Roy. Soc., A, Vol. 93, p. 1 (1916); Phil. Trans., A, Vol. 217, p. 157 (1917); Phil. Mag., Vol. 34, p. 146 (1917); Phil. Mag., Vol. 38, p. 182 (1919); Enskog, D., Arkiv. f. Nat. Astron.-o-Fysik., Stockholm XVI (1921); Chapman, S., and Hainsworth, W., Phil. Mag., Vol. 48, p. 602 (1924).

† Chapman, S., and Dootson, F. W., Phil. Mag., Vol. 33, p. 268 (1917); Ibbs, T. L., with a note by Chapman, S., Proc. Roy. Soc., A, Vol. 99, p. 385 (1921); Ibbs, T. L., Proc. Roy. Soc., A, Vol. 107, p. 470 (1925); Elliott, G. A., and Masson, I., Proc. Roy. Soc., A, Vol. 108, p. 378 (1925).

Eleven pairs of gases were examined in the course of the experiments. These can be grouped as follows for purposes of comparison: Carbon monoxide and nitrogen were each examined with hydrogen, carbon dioxide and nitrous oxide as subsidiary gases. Carbon dioxide and nitrous oxide were each examined with hydrogen, oxygen, nitrogen and carbon monoxide as subsidiary gases. Four of the pairs can thus be used for the purposes of each comparison. Mixtures of ethylene and hydrogen were also examined in order to compare the behaviour of ethylene with that of carbon monoxide and nitrogen. This comparison is of interest, as the molecules C_2H_4, CO and N_2 have practically the same masses, but the C_2H_4 molecule differs from the others in size and in its number of atoms. The somewhat related problem of obtaining a partial separation by thermal diffusion in mixtures of nitrogen and ethylene has been discussed* by Professor Chapman.

METHOD OF EXPERIMENT.

The method of experiment was essentially the same as that employed in a previous series† of experiments, but a few modifications were introduced to meet the special needs of this work. A uniform mixture of two gases is introduced into the apparatus, which can be regarded as two vessels maintained at different temperatures joined by a connecting tube. A glass bulb *A* (Fig. 1), of volume 99·8 c.c., surrounded by a steam jacket *B*, served as the hot side. This was joined by a connecting tube *C*, of length 2·5 cm., and internal diameter 4 mm., to a brass cap *S* screwed to a katharometer block *K*. The screw cap and katharometer block were kept at constant temperature by means of a water jacket supplied from the mains. The total volume of the cold side, which includes the inside of the screw cap and the katharometer cell, was 1·39 c.c. The length of the connecting tube is reduced as far as possible, so that the temperature gradient is steep, and the time for the effect to take place is short. For mixtures containing hydrogen the effect could be regarded as complete in about four minutes, for other mixtures about ten minutes was allowed.

As all the gases used in these experiments could be obtained in considerable quantities, it was possible to make the hot side larger than before, with the result that nearly all the effect will show itself as a change on the cold side, where it is measured. The corresponding small effect on the hot side can be calculated. The thermal separation is expressed throughout as the difference in percentage composition by volume, between the gas on the two sides when equilibrium is reached. By making the hot side large we reduce the possibility of error due to uncertainty of the exact volumes of the connecting tube or cold side. For the purpose of making a comparison it was convenient to work with the hot side at steam temperature throughout. This facilitates the examination of a large number of mixtures requiring many observations. Except for the mixtures containing hydrogen, the mass ratio of the molecules is small, so that the maximum separation obtained with the hot side at about 100°C. and the cold side at about 10°C. will not be great. For these mixtures it was found to be less than 0·5 per cent. There is no advantage, however, in using a greater temperature range, as this tends to make the conditions of experiment less steady. The method adopted gave consistent results, and it was found that satisfactory measurements could thus be made on mixtures of small mass ratio.

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* Chapman, S., Phil. Mag., Vol. 34, p. 146 (1917).

† Ibbs, T. L., Proc. Roy. Soc., A, Vol. 107, p. 470 (1925).

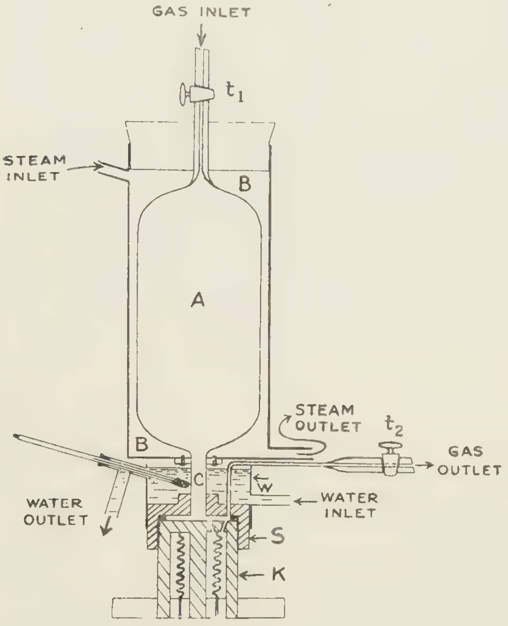

FIG. 1.—DIAGRAM OF APPARATUS.

In some of the experiments the gases were mixed in a cylinder under pressure. Steam was passed through the jacket, and the cold water turned on. The gas mixture was then allowed to flow through the apparatus, entering at tap t_1 and leaving at tap t_2. In this way the flow of gas through the connecting tube overcomes the tendency for thermal separation, and gas of very nearly uniform composition fills the apparatus. The flow is then stopped, taps t_1 and t_2 are closed, and thermal diffusion takes place, the change of composition of the gas on the cold side being measured by the katharometer. The temperature of the steam gives the hot side temperature T_1, and the temperature of the cold water gives the cold side temperature T_2.

In other cases, where it was undesirable to mix the gases under pressure in a cylinder, the following rather different procedure was adopted. The gases were first passed separately into the apparatus, and allowed to mix by diffusion at atmospheric pressure. Steam was then passed through the heater. If the taps were kept closed this would result in a change of pressure, which would itself affect the katharometer readings. To avoid any difficulty in making a correction for this, tap t_2 is opened when the heater is applied. Some gas, therefore, is driven out from the hot side through the connecting tube and atmospheric pressure is maintained. By applying the steam suddenly to the heater the gas escapes before much thermal separation has taken place. The loss of this gas, therefore, does not make any serious change in the mean composition of the gas in the apparatus. The glass bulb A is at the temperature of the room before the steam is applied, and this may differ by a few degrees from the temperature of the cold water. This will result in a small initial difference in composition between the gas on the two sides due to the thermal diffusion effect. What is actually measured when the steam is applied is the further change in composition which takes place on the cold side. In this case T_2 can be regarded as the temperature of the glass bulb before the steam is applied, and T_1 the temperature after the steam is applied. Special experiments were made which showed that this method and the flow method previously mentioned gave concordant results.

With the hot side at 100°C. and the cold side at 10°C., the value of $\log_{10}\frac{T_1}{T_2}$ is nearly 0·12, T_1 and T_2 being expressed in absolute measure. For the purposes of comparison it has been convenient to work out all results for $\log_{10}\frac{T_1}{T_2}=0{\cdot}12$. No serious error can be introduced by assuming for all mixtures that the separation is proportional to $\log_{10}\frac{T_1}{T_2}$ for the temperature ranges actually used.

The bridge arrangement of the katharometer was similar to that previously adopted.* The instrument is used for two distinct purposes : (*a*) for the analysis of gas mixtures, accurate to about 0·5 per cent. ; (*b*) for the measurement of the small changes in composition produced by thermal diffusion, with a probable accuracy of about 0·005 per cent. Eight new calibration curves were required. These were made by means of apparatus designed for making experiments on small quantities of gases, which it is hoped to describe later. A number of mixtures of known composition were made for each pair of gases by using a constant volume gas burette of the type described by Travers.†

We are indebted to Dr. G. A. Shakespear for his kindness in lending the katharometer used in this work.

PURITY OF GASES.

As this method of experiment requires considerable quantities of gases, it was convenient to use them as supplied commercially in cylinders under pressure. These gases could generally be regarded as sufficiently pure for the purpose. A satisfactory way of examining the purity is by testing the gas from the supply cylinder without admixture for thermal diffusion in the usual way. A small amount of impurity having about the same molecular weight as the gas will have little effect, whereas the same proportion by volume of an impurity of considerably different molecular weight may produce a greater effect which cannot be neglected. The nitrogen, carbon monoxide and hydrogen were practically pure, and the carbon dioxide and nitrous oxide contained only small amounts of impurity. We are indebted to Prof. F. W. Burstall, the Vice-Principal of the University, for providing a quantity of carbon monoxide of a high degree of purity. This was made in the plant in his department, and the gift was particularly welcome as the gas cannot be obtained commercially in a pure state. The oxygen contained about 3·0 per cent. of argon as an impurity. In mixtures of this gas with carbon dioxide or nitrous oxide the effect of the impurity would not be serious, previous experiments having shown that argon and carbon dioxide behave somewhat similarly. The ethylene contained about 1·0 to 1·5 per cent. of hydrogen. This impurity is serious, except in mixtures of the gas with hydrogen, and only measurements on this pair are recorded.

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* l.c., p. 475.

† Travers, M. W., " Study of Gases," p. 69.

RESULTS.

The results for the eleven pairs of gases are shown in Tables I, II and III. All are expressed as the percentage separation obtained when $\log_{10}\frac{T_1}{T_2}=0\cdot12$. The percentage of the lighter gas in the original mixture is stated throughout. The results

TABLE I.

Nitrogen and Carbon dioxide.		Carbon monoxide and Carbon dioxide.		Nitrogen and Nitrous oxide.		Carbon monoxide and Nitrous oxide.	
Per cent. N_2 in mixture.	Separation per cent.	Per cent. CO in mixture.	Separation per cent.	Per cent. N_2 in mixture.	Separation per cent.	Per cent. CO in mixture.	Separation per cent.
3·75	0·044	11·8	0·132	16·3	0·160	11·4	0·114
12·5	0·133	26·4	0·269	24·4	0·218	23·0	0·205
17·6	0·185	32·1	0·334	34·7	0·309	33·3	0·318
26·9	0·282	39·9	0·403	39·3	0·342	40·8	0·363
35·3	0·366	44·6	0·402	41·0	0·352	47·8	0·388
40·6	0·394	50·25	0·415	41·0	0·362	51·7	0·397
44·9	0·407	51·2	0·428	45·6	0·387	55·3	0·398
45·1	0·401	59·8	0·401	49·0	0·394	62·0	0·352
49·8	0·421	67·8	0·341	54·8	0·384	68·0	0·312
57·0	0·409	77·0	0·261	56·6	0·381	80·7	0·199
67·2	0·355	...	...	59·6	0·365	...	...
80·8	0·199	...	...	72·0	0·269	...	...
92·9	0·086	...	...	84·1	0·156	...	...

TABLE II.

Carbon dioxide and oxygen.		Nitrous oxide and oxygen.		Carbon dioxide and hydrogen.		Nitrous oxide and hydrogen.	
Per cent. O_2 in mixture.	Separation per cent.	Per cent. O_2 in mixture.	Separation per cent.	Per cent. H_2 in mixture.	Separation per cent.	Per cent. H_2 in mixture.	Separation per cent.
20·0	0·203	17·1	0·157	3·5	0·158	2·0	0·119
32·9	0·321	23·1	0·244	8·4	0·433	9·4	0·445
39·8	0·364	27·0	0·281	13·6	0·683	11·1	0·573
43·3	0·383	33·3	0·345	15·7	0·831	12·5	0·605
45·5	0·396	35·0	0·377	19·2	0·95	18·7	0·930
47·0	0·383	38·6	0·376	22·2	1·15	25·6	1·24
49·4	0·400	42·4	0·386	23·0	1·18	31·0	1·44
53·8	0·392	46·1	0·393	24·4	1·21	36·4	1·58
55·8	0·391	47·9	0·391	28·3	1·43	38·1	1·63
57·3	0·385	50·0	0·407	32·8	1·56	40·8	1·71
58·5	0·388	52·9	0·397	34·0	1·54	43·9	1·83
61·1	0·372	57·9	0·395	37·1	1·74	50·1	1·93
68·7	0·323	64·5	0·369	42·1	1·94	53·2	1·99
76·6	0·282	80·0	0·249	46·7	1·99	61·0	2·07
79·6	0·292	85·7	0·203	63·5	2·18	69·0	2·02
87·5	0·204	93·8	0·128	71·4	2·10	72·7	1·97
90·4	0·170	...	...	78·0	1·95	78·6	1·88
...	...	...	...	80·0	1·83	83·5	1·52

TABLE III.

Carbon monoxide and hydrogen.		Nitrogen and hydrogen.		Ethylene and hydrogen.	
Per cent. H_2 in mixture.	Separation per cent.	Per cent. H_2 in mixture.	Separation per cent.	Per cent. H_2 in mixture.	Separation per cent.
5·5	0·282	5·2	0·264	5·4	0·242
8·0	0·405	9·6	0·451	10·5	0·466
15·7	0·841	18·9	0·967	17·7	0·787
24·4	1·21	25·4	1·34	25·1	1·105
30·1	1·49	36·9	1·80	29·0	1·28
35·7	1·69	46·6	2·25	39·6	1·60
42·4	1·95	54·7	2·54	48·2	1·73
48·5	2·18	55·8	2·60	58·3	2·16
55·0	2·54	61·0	2·63	64·4	2·40
61·7	2·53	62·1	2·58	74·5	2·34
63·3	2·64	66·4	2·62	74·8	2·28
65·3	2·62	68·2	2·54	82·0	2·23
70·4	2·46	76·4	1·98	84·6	1·69
77·8	2·19	81·6	2·03	85·1	1·52
81·9	1·65	85·0	1·45	89·6	1·37
83·4	1·57	87·2	1·21	...	...

are also shown graphically in Figs. 2, 3, 4 and 5. These composition-separation curves show clearly how the amount of separation depends on the proportions of the gases in the mixtures. More than three hundred measurements of thermal diffusion were made. Many of the values given represent the mean of a number of observations on the same mixture. In addition, numerous experiments were made which served to check the accuracy of different portions of the work.

DISCUSSION OF RESULTS.

All the mixtures examined showed the thermal diffusion effect of the sign predicted by theory: in all cases there was a tendency for the heavier molecules to move to the cold side, and the lighter molecules to the hot side. Previous results showing the influence of the mass ratio of the molecules in a mixture on the shape of the composition-separation curves are confirmed. When the mass ratio is nearly unity the curves are nearly symmetrical, the maximum effect being given in a mixture containing about 50 per cent. of the two gases (Figs. 2 and 3). For mixtures containing hydrogen, in which the mass ratio of the molecules is greater, the curves are unsymmetrical, and the maximum occurs in a mixture considerably richer in hydrogen than in the other gas (Figs. 4 and 5).

NITROGEN COMPARED WITH CARBON MONOXIDE.

Examination of the results shows that the effect is very nearly the same in mixtures containing either nitrogen or carbon monoxide and a subsidiary gas. The graphs drawn in Fig. 2 illustrate this clearly, the upper curve representing the effect for mixtures of either nitrogen with carbon dioxide or carbon monoxide with carbon dioxide; and the lower curve for either mixtures of nitrogen with nitrous oxide or carbon monoxide with nitrous oxide. In Fig. 5 it will be seen that the effect is very nearly the same for mixtures of nitrogen with hydrogen as for

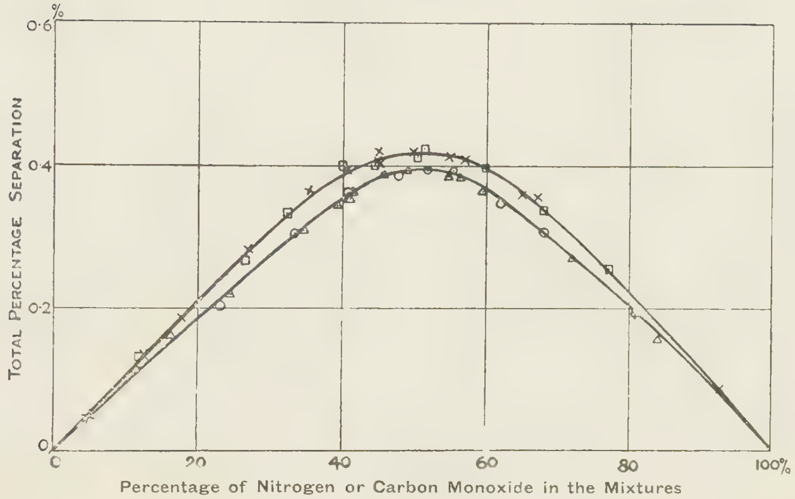

Fig. 2.

Composition—Separation Curves.

$\left[\mathrm{Log}_{10}\frac{T_1}{T_2}=0\cdot12\right]$

Upper Curve { 1. Nitrogen and Carbon Dioxide . . ×
2. Carbon Monoxide and Carbon Dioxide ⊡

Lower Curve { 3. Nitrogen and Nitrous Oxide . . . △
4. Carbon Monoxide and Nitrous Oxide ⊙

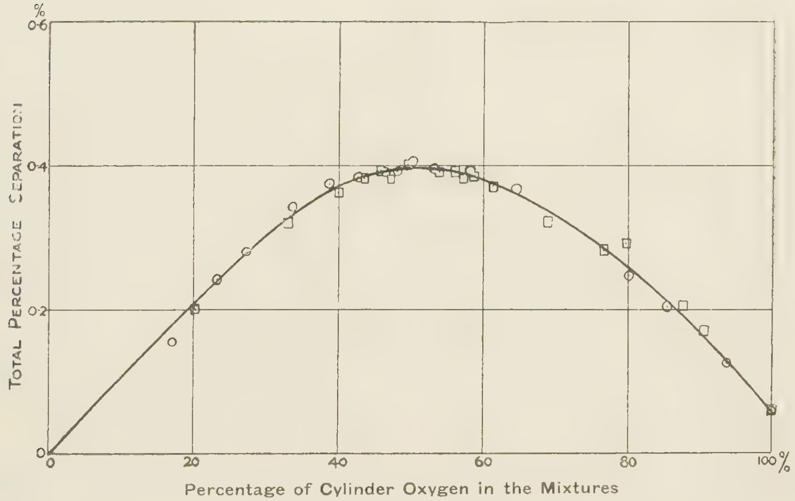

Fig. 3.

Composition—Separation Curves.

$\left[\mathrm{Log}_{10}\frac{T_1}{T_2}=0\cdot12\right]$

1. Oxygen and Carbon Dioxide . . . . . . ⊡
2. Oxygen and Nitrous Oxide . . . . ⊙

Note.—The effects shown by the two mixtures are practically the same. The separation shown at 100 per cent. is due to impurity in the Oxygen used.

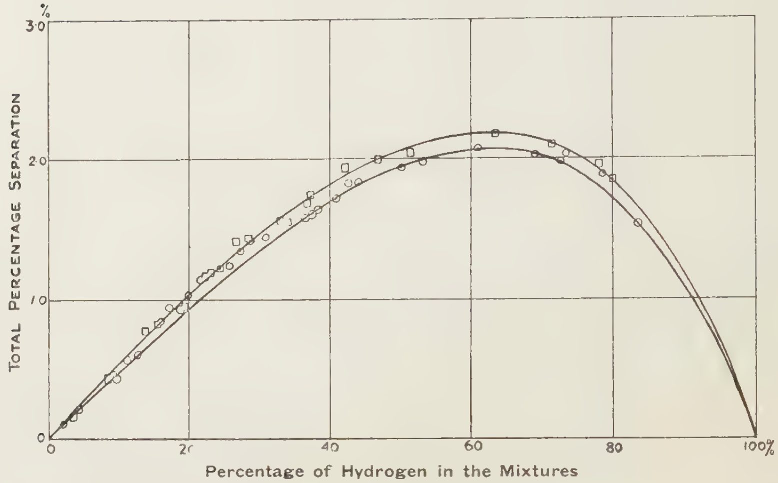

FIG. 4.

Composition—Separation Curves.

$$\left[\log_{10}\frac{T_1}{T_2}=0{\cdot}12\right]$$

1. Hydrogen and Carbon Dioxide . . . . . ⊡
2. Hydrogen and Nitrous Oxide . . . . . . ⊙

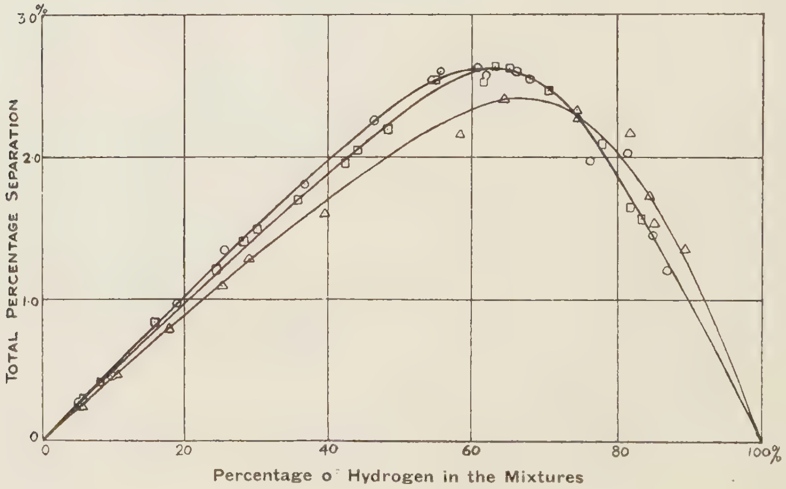

FIG. 5.

Composition—Separation Curves.

$$\left[\log_{10}\frac{T_1}{T_2}=0{\cdot}12\right]$$

1. Hydrogen and Carbon Monoxide . . . . . ⊙
2. Hydrogen and Nitrogen . . . . ⊡
3. Hydrogen and Ethylene . . . . △

mixtures of carbon monoxide with hydrogen, although these two curves cannot quite be superposed completely. It may be considered that the effects produced by nitrogen or carbon monoxide in any of the mixtures do not differ by more than the possible errors of these experiments. C. J. Smith* has made an accurate comparison of the viscosities of these two gases, using Rankine's method. Measurements made at 15°C. and 100°C. indicate that the viscous properties of the two gases are identical over this range of temperature. The mean collision area, deduced by using Chapman's formula, is given as $0{\cdot}767 \times 10^{-15}$ sq. cm. for both gases, so that the molecules can be regarded as having the same size. The similarity of the behaviour in thermal diffusion therefore indicates that the molecular fields must be closely alike. This is of particular interest, as there is known to be a similarity of structure in the two gases, both having the same number (14) of extra-nuclear electrons. In the Lewis-Langmuir theory of molecular structure the two gases are considered to have the same electronic arrangement, and Langmuir† attaches importance to the idea that these gases can be regarded as examples of a special case where the whole molecule lies inside one cube formed by eight electrons. The fact that the two gases have very similar physical properties is in keeping with the idea of the similarity of their structure and external fields. The work of Bone‡ and his collaborators on the "activation" of nitrogen appears to indicate a further relationship between the two gases. Information given by viscosity measurements§ on nitrogen and carbon monoxide had previously indicated that their molecular fields are alike. Viscosity in a gas is, however, concerned with the forces arising between molecules of the same kind, while thermal diffusion is due to collisions between molecules of different kinds. The consistency of the behaviour of these two gases, of very different chemical properties, when in collisions of the latter kind appears to be of interest.

ETHYLENE.

The separation obtained in mixtures of ethylene and hydrogen is less than that in mixtures of nitrogen or carbon monoxide and hydrogen, as shown by the curve in Fig. 5. The mass ratio of the molecules in these mixtures is the same, but the size of the ethylene molecule is greater than that of carbon monoxide or nitrogen. The increased size of the heavier molecule which we have in ethylene-hydrogen mixtures would itself tend to increase the separation. The C_2H_4 molecule must therefore be regarded as softer than the molecule N_2 or CO. The curve for ethylene-hydrogen shown in Fig. 5 differs from the other two in shape, being more unsymmetrical. As the ethylene used in these experiments was impure, too much importance must not be attached to this difference in shape at present.

NITROUS OXIDE COMPARED WITH CARBON DIOXIDE.

A comparison of the behaviour of nitrous oxide and carbon dioxide in mixtures with subsidiary gases shows that the effects produced by the two gases are nearly the same. Fig. 3 shows that the effect for nitrous oxide and oxygen differs little from that for carbon dioxide and oxygen. In all other cases the effect given by nitrous oxide and a subsidiary gas is rather less than that given by carbon dioxide. Thus, in Fig. 2, the curve for carbon dioxide with either nitrogen or carbon monoxide lies above the curve for nitrous oxide with either nitrogen or carbon monoxide, and in Fig. 4 the curve for carbon dioxide and hydrogen lies above the curve for nitrous oxide and hydrogen. The work of C. J. Smith* has shown that the viscous properties of carbon dioxide and nitrous oxide are very nearly the same at 15°C. and 100°C., and he deduces that the mean collision area of the molecules of each of these gases is $0{\cdot}834 \times 10^{-15}$ sq. cm. These two gases of equal molecular weight and size are also similar in their physical properties, although in this respect they are not as closely alike as nitrogen and carbon monoxide. Again there is a similarity of structure, both gases having the same number (22) of extra-nuclear electrons. Langmuir considers that both have the same electronic arrangement : the outer electrons forming three cubes in line, the middle cube sharing two of its faces with the outer ones. An atomic nucleus lies in each cube. Rankine† has discussed the similarity of these two kinds of molecules. The structure suggested for the two gases is, however, not as compact as that suggested for nitrogen and carbon monoxide, where both nuclei are supposed to lie inside one electronic cube. The gases will differ in properties which depend on the distribution of the atomic nuclei in the molecules. We may expect a difference in the molecular moments of inertia ; and although the total charge on the three nuclei is the same in both cases, we may expect some difference in the mean external field. These measurements show that there is a small difference in the fields, and that the CO_2 molecule must be a little harder than the N_2O molecule. Viscosity measurements‡ have previously shown that the fields do not differ greatly, but indicated that the N_2O molecule was rather harder than the CO_2 molecule.

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* Smith, C. J., Proc. Phys. Soc., Vol. 34, Part 4, p. 155 (1922).

† Langmuir, I., Amer. Chem. Soc. Journal, Vol. 41, I, p. 868 (1919).

‡ Bone, W. A., and Haward, W. A., Proc. Roy. Soc., A, Vol. 100, p. 69 (1922) ; Bone, W. A., Newitt, D. M., and Townend, D. T. A., Proc. Roy. Soc., A., Vol. 103, p. 205 (1923).

§ cf. Jeans, J. H., Dynamical Theory of Gases, 3rd edit. p. 284.

The gases examined in these experiments provide a special and simple case for the application of the theory. A direct comparison of the thermal separation obtained in the mixtures gives information of a comparative nature on the molecular fields of certain gases. It also indicates a relationship between the external field and the molecular structure. The possibility of applying thermal diffusion to the investigation of such problems has been pointed out by Prof. Chapman. No attempt will be made here to compare the separation actually obtained with that predicted by theory. The results of the experiments have been communicated to Prof. Chapman, and it is hoped that he and Mr. Hainsworth will examine more fully their general bearing on the question of inter-molecular fields.

In the course of this work measurements were made on three pairs of gases which had previously been examined by one of the writers§—hydrogen and carbon dioxide, nitrogen and carbon dioxide, and hydrogen and nitrogen. For hydrogen and carbon dioxide the shape of the composition-separation curve and the amount of the effect are in agreement with previous results obtained by this method. The separation is, however, relatively less than that obtained by Elliott and Masson‖ when making experiments with the hot side at 490°C. The symmetry of the composition-separation curve for nitrogen and carbon dioxide, which was previously noted, is confirmed, many more observations being made in this case. The modified method employed gives steady conditions, which tend to give increased accuracy. The maximum separation obtained for hydrogen and nitrogen is relatively rather greater than that previously recorded.

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* l.c., p. 159.

† Rankine, A. O., Proc. Roy. Soc., A, Vol. 98, p. 369 (1920).

‡ cf., Jeans, J. H., l.c.

§ cf., l.c.

‖ Proc. Roy. Soc., A, Vol. 108, p. 378 (1925).

In conclusion, we desire to express our thanks to Prof. S. W. J. Smith for the help he has given by granting full facilities for the work.

DISCUSSION.

Prof. S. CHAPMAN said that the molecular similarity of the gases discussed, which had already been shown by Rankine and Smith, was still more directly proved by the present method. Thermal diffusion is a phenomenon discovered by means of mathematical analysis, and it is very difficult to form any physical picture of the way in which it arises: wherever a temperature gradient exists in a mixture of gases, a separation of the components takes place until equilibrium with the mixing effect of ordinary diffusion is established. The phenomenon disappears when the attraction between molecules varies as r^{-5}, and its magnitude affords a measure of $(n-5)$ when the attraction varies as r^{-n}. Every point on the curves obtained by the authors gives a measure of n, whereas in the viscosity experiments n has to be inferred from the viscosity at two temperatures, from the formula $\log v = \{(n+3)/2(n-1)\} \log T$, where v is the viscosity and T the temperature. Further, the coefficient of $\log T$ is not very sensitive to variations of n, so that the present method affords a far more delicate measure. On the assumption that n is 2 in stars where ionisation is complete, the quantity $(n-5)$ would be negative, and the effect of thermal diffusion would be reversed in sign, the heavier molecules moving towards the higher temperature.

Prof. A. O. RANKINE said that he had seen the authors' work while it was in progress, and was struck by the amount of labour involved, which was much greater than appeared from the Paper, since it necessitated precise variations in the percentages of the mixtures used.

Dr. EZER GRIFFITHS: There is just one question I should like to ask the authors. It concerns the arrangement of their apparatus. From the point of view of accuracy in the observations, would it not have been more satisfactory if the hot wire katharometer had been arranged so that one of the gaseous mixtures had access to one of the platinum spirals, and the other gaseous mixture to the other spiral? The observed quantity would then be the " difference " in the properties of the two mixtures, which is what the authors are seeking. Such an arrangement would have the added convenience that the two mixtures could be subjected to identical temperature conditions and interchanged from one side to the other to eliminate instrumental error.

AUTHORS' reply: A differential method as suggested by Dr. Ezer Griffiths, where gas mixtures have access to both platinum spirals, has already proved to be of value for a number of purposes, in which the measurement of small differences is involved. Its use however in the type of thermal diffusion measurement described in the Paper, appears to present certain additional difficulties in the arrangement of the apparatus, and in making the measurements. In addition to observations of a comparative nature on the different pairs of gases, measurements of the actual amounts of separation produced in the gas mixtures were required.

XVIII.—RELIGHTING OF A NEON LAMP WHEN MOMENTARILY EXTINGUISHED AT VOLTAGES BELOW THE STRIKING POTENTIAL.

By Robert R. Nimmo, *M.Sc.*, Assistant to the Professor of Physics, University of Otago, Dunedin, New Zealand.

Received November 27, 1926.

(*Communicated* by Prof. R. Jack.)

Summary.

The Paper gives an account of an investigation on the time for which the continuous discharge of a neon lamp may be interrupted without putting out the lamp. This time, which has been found to be of the order of 50 micro-seconds, depends on the voltage across the lamp and also on the current passing through it.

Introduction.

A NEON lamp discharge starts spontaneously if the striking voltage is applied to the lamp, but it would conceivably be possible to start the discharge at a lower voltage than this if a large supply of ions were initially present in the lamp. If a discharge be made to pass through the lamp and then interrupted for a short time, the supply of ions which was present when the discharge was passing will quickly diminish. If at the end of this short interruption a voltage smaller than the striking voltage be applied to the lamp, the discharge may recommence. It is of interest, then, to investigate the maximum time for which a lamp discharge may be interrupted so that sufficient ions are left in the lamp to allow the discharge to recommence, when the voltage applied to the lamp is always below the striking voltage.

This problem has been investigated by Oschwald and Tarrant.* Although they interrupted the lamp discharge for periods as short as 50 micro-seconds, the lamp never relit even when voltages very little below the striking voltage were used.

Experimental.

The arrangement used in interrupting the lamp discharge is illustrated in Fig. 1. The lamp used was an *I* neon lamp not previously overrun.† In series with the lamp was a resistance box *R* and a galvanometer shunt *G*. The switch *K* was of the rotating type, and was employed to connect an uncharged condenser in parallel with the lamp. Later in the rotation of the switch the condenser was discharged through the resistance *m*. When the condenser charges through the resistance *R* it can readily be shown that the voltage across the neon lamp is below the extinction voltage E_1 for a time‡ $RC \log_e \frac{E_0}{E_0 - E_1}$, where E_0 is the battery voltage and *R*§ and *C* the values of the resistance and capacity. The rest of the experimental arrangements were as follows. When the switch *Z* was open the condenser was already charged when connected across the lamp, and so opening *Z* really put the rotating switch *K* out of action; hence *K* could be started or stopped without the trouble of starting or stopping the motor driving it. One section (30 volts) of the accumulator battery was put on charge during the experiments, and by varying the charge rate the voltage across the lamp could be adjusted. The voltage used had to be below the striking voltage for the lamp, and so the switch *S* and the top part of the battery were used to light the lamp. The 10 mfd. condenser shown was important, for its charge kept the lamp alight while the switch *S* was put from "*a*" to "*b*," and also, owing to its large value compared with that of the condenser *C* it greatly minimised the effect that would be produced by the resistance of the battery. The condenser *C*, which in charging interrupted the lamp discharge, was a $\frac{1}{3}$ mfd. standard mica condenser.

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* Oschwald and Tarrant, Proc. Phys. Soc., Vol. 36, p. 262.

† The ballast resistance was of course removed.

‡ Some assumptions made in deducing this time are examined below.

§ For reasons given below, *R* does not include the battery resistance if this is small.

In the actual experiments, *Z* was opened, the lamp was lit and *S* was put to "*b*." The switch *K* had previously been started, and when *Z* was closed the interruption commenced. The voltage applied to the lamp and resistance *R* was kept constant, and, as usually happens, the current through the lamp gradually fell with time. It was on this account that a rotating switch was used. It was found that when the current passing through the lamp was large the charging of the condenser did not extinguish the lamp when *R* was, say, 100 ohms; but later, when the current had dropped considerably, the lamp discharge was extinguished on charging the condenser through 90 ohms. By using a rotating switch the lamp discharge was interrupted several times while the lamp current was approximately constant, and so it was possible to give *R* a definite value and go on interrupting the discharge until the current fell to such a value that the interruption was sufficient to extinguish the lamp. Thus the galvanometer reading gradually fell with time, and then suddenly it dropped to zero. The reading of the galvanometer was taken just before the sudden drop to zero, and this gave the lamp current for which the extinction interval used was just large enough to cause interruption of the discharge. The galvanometer reading just before extinction, the applied voltage when the current was flowing and the resistance used were all noted. The galvanometer reading just before extinction was somewhat variable, and so about ten values were taken with the same values of *R*, *C* and the applied E.M.F. The means of these sets are given in the table on next page.

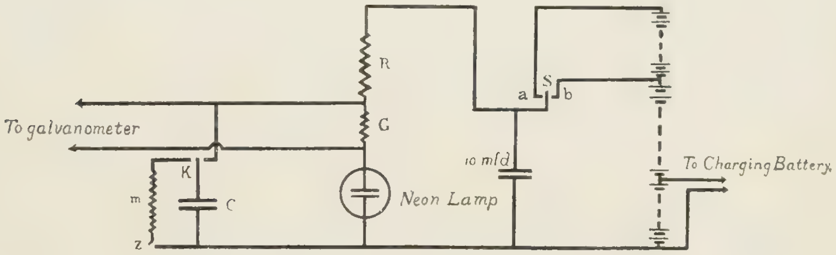

FIG. 1.

The table below shows the maximum time for which the lamp discharge could be interrupted without extinction occurring when the voltage* and current had the values stated. The time of interruption of the discharge was calculated on the assumption that interruption occurred when the lamp potential was below 142 volts, the extinction voltage for a continuous discharge. One lamp only was experimented upon.

RESULTS.

Voltage across lamp. Volts.	Current at extinction. Milliamps.	Resistance. Ohms.	Time of interruption. Microsecs.
170	6·5	110	66
	6·3	100	60
	5·9	90	54
	5·7	82	49
168	6·8	100	62
	6·3	90	56
	5·9	80	50
	5·5	70	44
166	6·5	80	52
	5·8	70	45
	5·3	65	42
164	6·2	70	47
	5·8	65	43
	5·6	60	40
	5·0	55	37
	4·9	53	35
162	6·0	63	44
	5·7	60	42
	5·2	55	38
	4·8	50	35
160	5·8	60	44
	5·4	55	40
	4·8	50	36
	4·5	45	33
158	4·6	45	37
	4·2	40	33
156	4·0	35	28
154	3·7	30	26
152	3·3	25	23
150	3·0	20	20

The first voltage used (170 volts) was very close to the striking voltage, possibly above it, for when this voltage was applied the lamp struck after about 30 seconds.

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* In giving the value of the voltage across the lamp no deduction has been made for the small potential drop across the resistance R in series with the lamp.

The lamp discharge during interruption was examined in a rotating mirror, and in general two very fine black lines were seen across the red image in the mirror. This was not the case when lamp currents less than 1 milliamp. were used, for then the mirror showed that the second interruption occurred before the lamp had recovered from the effect of the first. From the evidence afforded by the rotating mirror, it is considered that successive discharges had no cumulative effect in the cases for which values are given.

Discussion of Method and Results.

It was pointed out above that some assumptions were made in deriving the period of interruption. In the first place non-inductive resistances are assumed, but if properly wound resistance coils are used the inductance present will produce a negligible effect.

A considerable error might be caused by contact resistance at the switch. As care was taken to see that a good contact was made between the brush and the rotating brass segment at the switch, it is considered that errors due to contact resistance should be small.

One point of interest is the fact that the maximum interruption interval, which does not extinguish the lamp, depends to a considerable extent on the current passing. For a given voltage applied to the lamp this maximum interruption interval is much bigger for large currents than for small ones, as is to be expected. For a given voltage applied to the lamp, if below 166 volts, the ratio of lamp current to maximum interruption interval is approximately constant (within 4 per cent.) ; but for higher voltages applied to the lamp this ratio is found to be smaller for large currents than for small ones.

In conclusion, I wish to thank the British General Electric Company for the gift of a number of *I* neon lamps, and Dr. Jack and rest of the staff of the Physics Department for consideration which they have shown me and which has been of great value in enabling me to carry out this work.

DISCUSSION.

Mr. A. G. **Tarrant** said that he and Mr. Oschwald had recently measured not only the minimum time of interruption, but the actual voltage at relighting for interruptions down to 3 micro-seconds, and had found no case where lighting took place below the upper critical voltage. He suggested that in the author's apparatus surges of current may take place, owing to the presence of inductance in instruments and leads, and so raise the starting voltage above the applied D.C. voltage. This view was consistent with the effect of varying R as found by the author. Further, the author deduced the duration of the interruption from an expression involving the difference between two large voltages not accurately known. It would be of interest for the author and the speaker to experiment on one another's lamps, in order to eliminate divergences of result due to differences between the lamps.

Mr. R. P. **Fuge** said that he had measured the time taken to light up for various voltages lying between the upper and lower critical voltages, the lamps being allowed to rest in the dark for half an hour with their terminals short-circuited after each experiment. The time taken at a given voltage by a given lamp varied irregularly between astonishingly wide limits, but the averages for a number of trials lay on a smooth curve, part of which was accurately linear.

Dr. D. **Owen** considered that the principle of the author's method was sound. He had taken the opportunity to check the tests with a simplification of circuit, replacing the commutator by a simple mercury throw-in switch, and omitting the large condenser across the battery, the use of which by the author seemed open to question. In the case of the particular lamp used, the striking voltage was 145, the extinction voltage, 120 ; the E.M.F. of the battery after striking was reduced to 140 volts. Observations were made with a series of values of the capacity C

shunting the lamp, over the range 0.05 μF. to 1 μF., the corresponding values of the maximum series resistances R (from 20 ohms to 450 ohms) being found which just permitted of relighting of the lamp. On plotting the values of R against I/C a straight line was obtained, the intercept on the axis of R being -17 ohms, which agrees reasonably well with the ascertained value of 21 ohms found for the internal resistance of the battery. Using the formula of the Paper this gives the value 45×10^{-6} seconds for the period for which the lamp was below extinction voltage without failing to relight. This confirms the author's result ; and considering the wide range of capacities taken, it seems difficult to admit the suggestion of another speaker that the relighting might be assigned to voltage surges which lift the voltage across the lamp, above the striking voltage.

AUTHOR'S reply (communicated) : I would point that the supply circuit in which inductance might be expected, was quite free from instruments, and so any inductance in the leads would produce only a negligible effect. In any case similar results were obtained when no charging circuit was used, and in such circumstances surges could not have occurred. The difference between the two sets of results was possibly caused by the difference in the methods of extinction in the two cases, and not by errors in the experimental methods. I must admit that the derivation of the time of extinction is open to objection, but I would point out that, as the lamp certainly would not relight below the extinction voltage, the times of extinction given might be too small, and so the correct times would disagree still further with the results of Mr. Tarrant. In conclusion I may mention that there is at present no satisfactory way of defining the exact point at which the lamp relights much less of measuring it.

XIX.—ELECTRICITY OF DUST CLOUDS.—PART I.

By G. B. Deodhar, *M.Sc.*, Physics Department, University of Allahabad (India).

Received December 16, 1926.

(*Communicated by* D. B. Deodhar, *M.Sc.*)

Summary.

1. Observations on electrification of various dusts have been made by using a large volume of air at a pretty high speed. The climatic conditions are totally different from those under which other similar observations were made.

2. The number of substances tried is considerably increased. In some cases the sign of electrification is the same as that observed by Rudge.

3. The factors governing the phenomena of electricity of dust storms are stated as follows: (1) Material of the dust. (2) Its size. (3) The gas raising the cloud. (4) The velocity of the gas. (5) The temperature. Account of the first two factors is given in the present communication.

4. It appears that the electricity developed is of frictional nature, which itself is, of course, unknown.

5. Some quantitative estimates of electrification ot chlorides and nitrates of sodium and potassium are made. It is seen that chlorides of sodium and potassium are equally efficacious, whilst sodium nitrate is about 4½ times as efficacious as potassium nitrate.

6. Dusts of various roughly uniform sizes were obtained with the help of a few graded sieves. The magnitudes of the various sizes were obtained after very laborious observations under microscope. It is shown graphically that, other things being the same, the number of volts developed by blowing increases very rapidly as the size grows less.

Introduction.

THE subject of the electricity developed when a cloud of dust is raised by wind is of immense theoretical and practical importance, especially in the tropical regions like India, where large volumes of dust are blown up into the atmosphere, causing what are called dust storms. Such storms are very common in the months of April, May and June in the northern tracts of this country, and cause a great increase in the potential gradient. For the wireless operator the dust storms are a great menace, and demand his closest attention. For a laboratory worker the dust is not less annoying, since it interferes to a large extent with his work on electrometers and other things where electric leakage is to be avoided. It is thus essential to collect as much information on the electricity of dust clouds as possible, and work inside as well as outside the laboratory must be vigorously prosecuted. The factors coming into play in the production of electric charges during the raising of dust clouds by winds may be stated as follows: (1) The material of the dust, (2) its size, (3) the gas raising the cloud, (4) the wind velocity, (5) the temperature.

Exhaustive work is necessary to find out the exact nature of the influence of each of these factors. Let us now see what knowledge we possess over these several factors. W. A. Douglas Rudge has done interesting experiments on the nature of the electricity developed by raising a cloud of dust.(1 and 2) He has examined several kinds of dust, and is of opinion that "Acidic" bodies, such as finely divided silica or molybdic acid, give to the dust a positive charge, whilst basic bodies, such as lead oxide, or organic dusts, such as flour or coal, give a negative charge. Of course, to establish such a generalisation it is necessary to examine a very large number of substances, and so enough work should be done in this direction. As regards factor (2) it has been pointed out that in general for a given material fine dust produces a very large electrification. But no quantitative work is done in this direction. It appears that the remaining factors have not been definitely studied, and so information must be gathered on each of them before a satisfactory theory of the electricity of dust clouds is formulated.

I propose to give in a series of Papers an account of the exact influence of the size of dust particles, of the gas raising the cloud, of the velocity of the gas, and of the temperature of the enclosure in which the dust is blown. As a preliminary I propose to give in the present communication an account of my observations on the electricity of dust clouds and dust storms.

APPARATUS.

The apparatus used is quite different from that of Rudge, and the experimental conditions are also new. Instead of concentrated air, large volume is used, and the wind velocity is also pretty high (about 8 metres per second). The experimental conditions thus partially approach those found in Nature. Fig. 1 shows the plan of the apparatus.

It has a close similarity with the apparatus used by Dr. G. C. Simpson in the study of electricity of thunderstorms.(3) *AA* is a big cylindrical vessel of iron, having the base as shown in the figure. On the top of the conical portion *TT* an ebonite ring is mounted, and on this is fixed a piece of fine wire gauze. On this gauze some quantity of dust is placed. The cylindrical vessel is covered by a wooden lid, *L*, the under surface of which is covered with tin-foil. At *G′* there is a hole in the lid. This is covered with wire gauze, having electrical contact with the tin-foil, which, in its turn, has contact with the vessel. The vessel *AA* is connected through a convenient key to a sensitive electrometer, which was worked at a sensibility of 420 mms./volt. The vessel is enclosed by a wire gauze screen, *SS*, which is earth-connected from several points. A strong current of air is obtained by a fan blower worked by a motor. The air current passes through the metallic pipe *B* into the conical portion *TT*, and thus the dust kept on the gauze *G* is blown up and collected in the vessel. The pipe *B* is, of course, kept free from the vessel. Any charge developed is given to the vessel, and is at once detected by the electrometer. The charge on the air can be obtained by suspending any suitable collector inside the vessel, and connecting it to a convenient electrometer. The present Paper gives an account of only the charge on the dust particles. The account of the charge on the air is reserved for a later Paper.

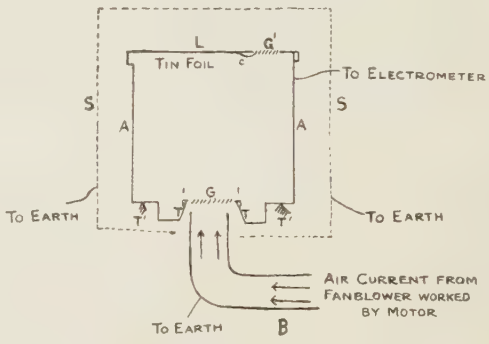

Fig. 1.—Dust Apparatus.

ELECTRIFICATION OF VARIOUS DUSTS.

Rudge has tried many substances ; but it was thought worth while to try many more kinds of dusts with an apparatus wholly different from that of Rudge, and under quite different climatic conditions. The temperature at the time of experiment was about 40°C. The electrometer was at first calibrated as usual. The sensibility was 420 mms./volt. The vessel was connected to the quadrants, and charged to a certain potential. Leakage was then observed without any air current passing into the vessel. Next, leakage with air current on was observed. The vessel with the connected quandrants was earthed, and then again kept insulated and uncharged. Air current was allowed to flow freely with full vigour. It was found that no electrical effect was produced. A small quantity of dust of a substance which was previously passed through two fine sieves was then carefully placed on the wire gauze *G*. The vessel and the quadrants earthed and then left insulated. Reading on the scale was noted. The dust was then blown by the air current and the reading of the electrometer again noted. After allowing for the leakage the difference between the two readings gave the electrification of the dust particles. For finding out simply the kind of electrification it was not necessary to find out the exact value in each case. The observations for various substances are collected in Table I below. For comparison Rudge's observations are also inserted.

TABLE I.

Substance.	Sign of electrification.		Remarks.
	Author.	Rudge.	
Ebonite dust	−	...	......
Rubber dust	+	...	When showered into the vessel (−).
Iron filings	−	−	Small.
Copper ,,	+	...	......
Brass ,,	+	...	......
Resin	+	...	......
Pulverised Lycopodium	+	...	......
Sulphur	−	...	......
Sealing wax	−	...	......
Precipitated chalk	−	−	......
Gulal (red)	...	...	......
Mica dust	(+), (−)	...	Sometimes (+), sometimes (−).
Road dust	+	...	......
Coal	+	...	......
Fine dust accumulating on things in laboratory	(+), (−)	...	Sometimes (+), sometimes (−).
Brick dust	+	...	Two pieces rubbed together both show feeble (+) charge.
Wheat flour	−	−	......
Gram	−	...	......
Coal ash	−	...	......
Wood coal	+	...	Two pieces rubbed against each other both gave (+) charge.
Glass	+	...	......
Rock salt	+	...	Two pieces rubbed against each other both showed (+) charge.
Zinc oxide	+	...	......
Turmeric	−	...	......
Pure $NaCl$	+	...	The salt must be pretty fine.
,, $NaNo_3$	+	...	......
,, KCl	+	...	......
,, KNO_3	+	...	......
,, NH_4Cl	+	...	......
,, $CaCl_2$	+	...	......
,, $BaCl_2$	+	...	......
,, Cobalt Chloride	+	...	Small. Difficult to determine, since the powder becomes moist very quickly.

SOME QUANTITATIVE DETERMINATIONS.

The observations made so far were all qualitative. It was next thought of making some quantitative determinations with a view to find out how the quantity of charge developed varies with the nature of the salt keeping other factors the same. The salts used were chemically pure. They were carefully pounded and passed through two fine sieves to obtain uniformity in size of the dust particles. Small quantity of each salt was measured by a sensitive balance, and carefully put on the gauze G, and the charge developed was measured as described above. Mean of several observations was taken as the charge developed by blowing a definite quantity of salt dust. A few observations are given in Table II :—

TABLE II.

Salt dust.	Mass.	Sign of charge.	Volts.
NaCl	5 grms.	+	1·13
$NaNO_3$	5 ,,	+	8·47
KCl	5 ,,	+	1·19
KNO_3	5 ,,	+	1·83

Of course, no claim is made for precision ; but it may be said with certainty that chlorides of potassium and sodium are equally efficacious, whilst potassium nitrate is very much less efficient than sodium nitrate. These observations are very few in number. Consequently no generalisation can be formulated. A more exhaustive study in this direction would be very interesting.

EFFECT OF SIZE.

Investigation on this point is beset with considerable amount of experimental difficulties. It is extremely difficult to secure particles of quite identical size. As a preliminary I have made some rough determinations on the electrification developed by raising a cloud of field dust, as this is expected to throw considerable light on the electricity of dust storms.

It is known for a long time that the presence of dust in the atmosphere has a very marked influence on the normal potential gradient. Rudge has made interesting observations in this connection. During a storm the magnitude of the gradient is considerably enhanced and the sign is either positive or negative, depending upon the nature of the dust.(4) I had no opportunity of determining quantitatively the potential gradient in a dust storm at Allahabad, which experiences heavy dust storms in summer (April, May and June) ; but a qualitative test of the high potential value is worth noting. During a dust storm a Geissler tube filled with helium was put in circuit with the aerial of our wireless set and the earth. It was found that the tube continued to glow quite brilliantly although intermittently. With the subsidence of the storm and settling of the dust the phenomenon vanished altogether, showing that during the storm the potential considerably exceeded the normal value. There is every reason to believe that the origin of the electricity of dust storms lies in the mutual friction between the dust particles blown up by wind. The dust storms at Allahabad are characterised by heavy winds and fine dust. I had no opportunity of measuring the wind velocity. A rough idea of the size of dust particles was obtained by allowing the dust to settle down at a sufficient height in a vessel, and observing the particles under microscope. The majority of the dust particles were below 0·01 mm. in size.

Next an attempt was made to get an idea of the way the electricity of dust particles varies with size. As remarked above the problem is pretty difficult. It is difficult to obtain a large number of dust particles of accurately uniform size. The following plan was therefore adopted to obtain particles, the size of which varied between certain limits which were not far between. Some quantity of dust was passed through various graded sieves, and thus the particles were classified in six different groups. Taking a weighed quantity for each group the charge developed due to blowing was determined as described in a previous section. The mean of several observations was taken in each case. A large number of particles belonging to a particular group was then observed under micrometer eyepiece, and by elaborate observations mean size of the particles deduced. The process was repeated for all the groups. The observations are given in Table III.

Table III.

Group.	Size.	Volts.
I.	3·15 mm.	1·40
II.	1·55 ,,	0·45
III.	0·86 ,,	0·35
IV	0·55 ,,	0·49
V	0·33 ,,	3·47
VI.	0·05 ,,	46·85

The whole process is, of course, very tedious, and gives only an approximate idea of the way in which the electrification of dust clouds varies with the size of the particles. Fig. 2 shows graphically how the charge varies with size.

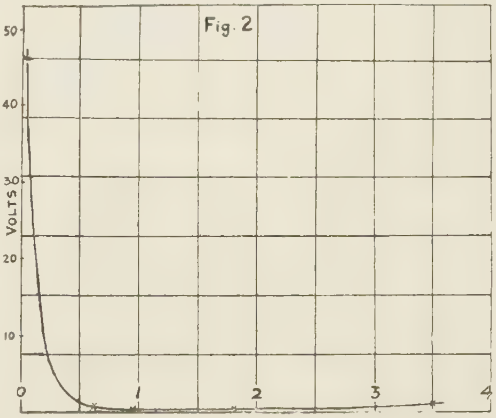
Fig. 2

It is seen at once from this figure that as the size decreases the volts produced mount up very high, thus explaining how the potential gradient mounts up considerably during storm or dust cloud raised by any means. Similar curve was obtained for barium chloride. It will be interesting to try many more substances in the same way.

As regards the remaining factors it appears very little is known. Work in this direction is in hand, and an account will be given in due course. During the course of the present observations it was incidentally observed that on halving the wind velocity almost the same electrification was obtained for the same quantity of dust.

Discussion of Observations.

It is easily seen from Table I that no definite rule can be given regarding the nature of electrification. Brass and copper, which are purely metallic dusts, differ from iron dust, but show similar electrification to various salts as well as some organic dusts. The electrification developed by friction between two pieces of a particular substance is also interesting, and an account is given in my note on anomalies in frictional electricity.(5) An examination of this note, as well as of Table I will bring out the fact that two pieces of the same substance can both be charged with the same sign. Obviously the dust abraded must be oppositely charged. This agrees with the observations of Hesecus,(6) and is in conflict with those of Rudge.

The quantitative work on salts is interesting, and it is contemplated to extend the observations on all possible chlorides. This will show how the electrification varies from group to group of the periodic table.

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DISCUSSION.

Dr. P. E. SHAW (communicated) : These experiments by Mr. Deodhar, and the well-known earlier ones by Mr. Rudge, have special interest for me, as I have recently sent a Paper bearing on this topic to the Society. The effects, however, which I obtain are due to the impact of only two like bodies, and in this way the condition of the impacting surfaces can be ascertained and varied at will. The dust cloud experiments are interesting, as they are closely analogous to the natural phenomena found in the open air under dry, tropical conditions ; but one may doubt if they are not altogether too complex to yield a solution of the problems involved. For instance, in these dust impacts the particles strike one another at all possible angles, and with a great range of speed. Such variables cannot be ignored. The curvatures of the striking surfaces must be very varied, and with the varied curvatures there will be different conditions of their adsorbed layers. Also, among the variables mentioned by Mr. Deodhar in the first clause of his Paper, the hygrometric state is not mentioned, though it is well known that moisture plays a very important rôle in tribo-electric effects.

Then, again, the charges observed in the apparatus shown are compounded, in unknown relations, of four parts :—

(1) Net charge, say $-ve$ left on the impacting particles ; (2) the contrary, say $+ve$, and equal, charge on the ions emitted to the air ; (3) charges of unknown sign and amount, due to impact of the particles on the walls and gauze of the chamber ; (4) ions, corresponding to these, emitted to the air.

True, a net charge from all these remains in the chamber, and is recorded on the electrometer when the air has been blown through it ; but how is it possible to disentangle the effects and eliminate those due to the chamber itself ?

Without anticipating the results of my own Paper, I may say that the electrical nature of surfaces may change fundamentally when rubbed or struck, so that it is possible to obtain a $+ve$ or $-ve$ charge at will from like solids. In a dust cloud we may expect some impacts to yield net $-ve$, some net $+ve$.

When the particles are very small, then, for the same total mass in a cloud, there would be correspondingly more contacts and greater charges, as Mr. Deodhar finds. How far the charges on, and motions of, colloidal particles, as in the Brownian movements, can be attributed solely to electrical charge arising by their impact, as in these experiments, one cannot say ; but there appear to be very interesting possibilities in this direction.

The molecular kinetic theory will account for two particles striking one another ; but their rebound would, in part, be due to repulsion of like charges, caused by impact.

Mr. J. H. COSTE (communicated): This Paper deals with a subject of considerable interest, not only from the meteorological point of view, but in relation to industrial powders, and probably to explosions due to organic dusts.

The author might find that he could grade his particles more easily by a process of sedimentation after sieving. For pure substances, in which the particles are of uniform density, sedimentation in liquids which have no solvent action, and which are relatively dense or/and viscous should be fairly satisfactory. It would probably be as well not to rely entirely on the calculation of particle size by Stokes' equation, but to verify the dimensions by the microscopical method, as Mr. Deodhar has done. Sedimentation is likely to be more trustworthy than elutriation.

A method of precipitation from solution might be found to yield particles more uniform in size and form than grinding does—e.g., NaCl can be precipitated from very dilute alcoholic solutions by dry gaseous HCl, yielding fine particles and other salts by adding to their solutions suitable liquids in which they are sparingly soluble.

The curve, Fig. 2, showing that voltage produced is an inverse function of particle size, lends support to the view that the change is directly related to the total surface on which it is distributed, and that with the velocities used a maximum change is obtained.

It would be interesting to know what precautions were taken to ensure the freedom of the air from dust before it reached the powder, and whether its humidity was determined or regulated.

DEMONSTRATION ILLUSTRATING SOME PHYSICAL PROBLEMS CONNECTED WITH APPLE TRANSPORT.

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A SKETCH was given of some of the problems which had been tackled in connexion with the refrigerated transport of fruit from overseas.

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The second problem was to provide means of removing the carbon dioxide produced by the fruit. An apple will generate its own volume of carbon dioxide in 40 hours if kept at a temperature of 70°F. and in 14 days if kept at 34°F. An experiment was shown to prove the generation of carbon dioxide by apples. The presence of carbon dioxide above 12 per cent. causes disease in the fruit. The suggested systems for supplying moisture-free air for ventilation were illustrated.

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CONTENTS.

PAGE

Portrait of O. W. Richardson, M.A., D.Sc., F.R.S., President 1926-28 *Frontispiece*

XIII. The Present State of Atomic Physics. (Presidential Address.) By O. W. Richardson, M.A., D.Sc., F.R.S. 171

XIV. Electrical Polarisation in Selenium Cells and the Effects of Desiccation. By A. O. Rankine, D.Sc., and J. W. Avery, B.Sc., D.I.C. 187

XV. Synchronous Alternating Current Motor and Mechanical Vibrating System Maintained by a Thermionic Valve at the Frequency of the Vibrating System. By T. G. Hodgkinson, M.C., A.M.I.E.E. 203

XVI. Measurements of Absorption Coefficients of Light Filters. By G. M. B. Dobson, D.Sc., and I. O. Griffith, M.A. 223

XVII. A Comparison of the Behaviour in Thermal Diffusion of Nitrogen and Carbon Monoxide, and of Nitrous Oxide and Carbon Dioxide. By T. L. Ibbs, M.C., Ph.D., and L. Underwood, M.Sc. 227

XVIII. Relighting of a Neon Lamp when Momentarily Extinguished at Voltages Below the Striking Potential. By Robert R. Nimmo, M.Sc. 238

XIX. Electricity of Dust Clouds.—Part I. By G. B. Deodhar, M.Sc. 243

Demonstration Illustrating Some Physical Problems Connected with Apple Transport. By Ezer Griffiths, D.Sc., F.R.S. 250